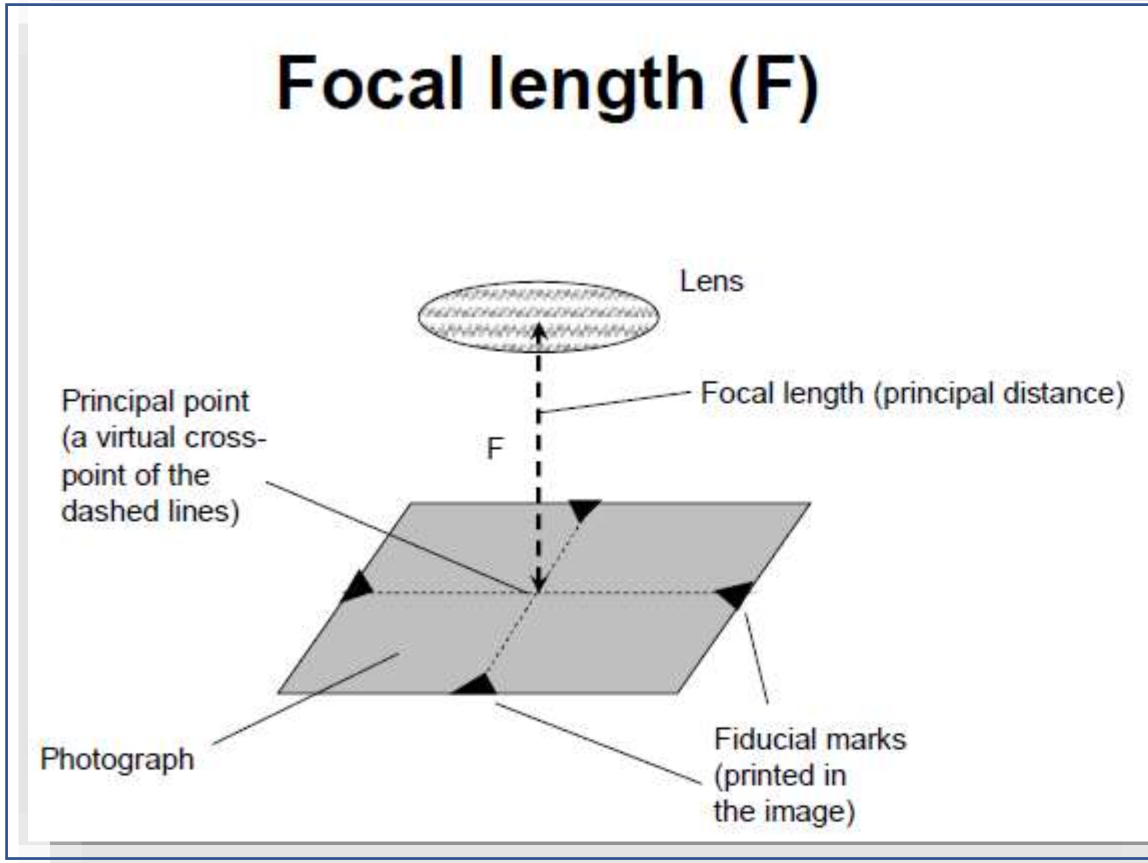
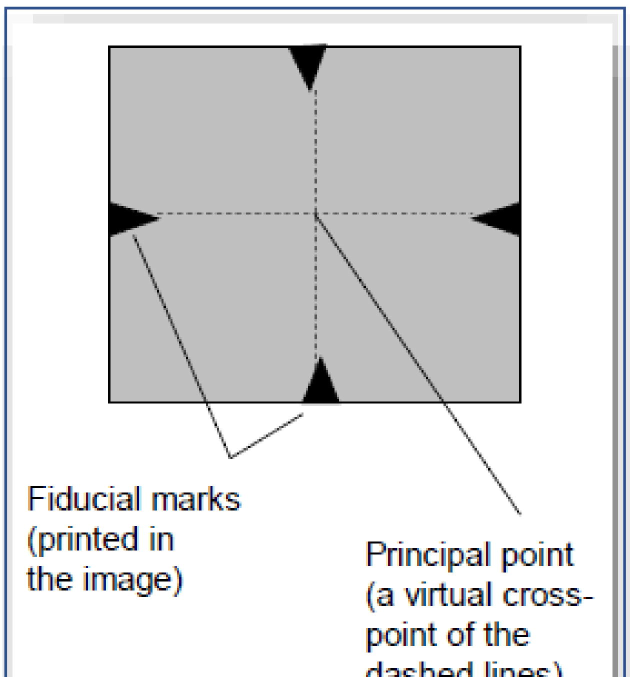
Geometry of Aerial Photographs

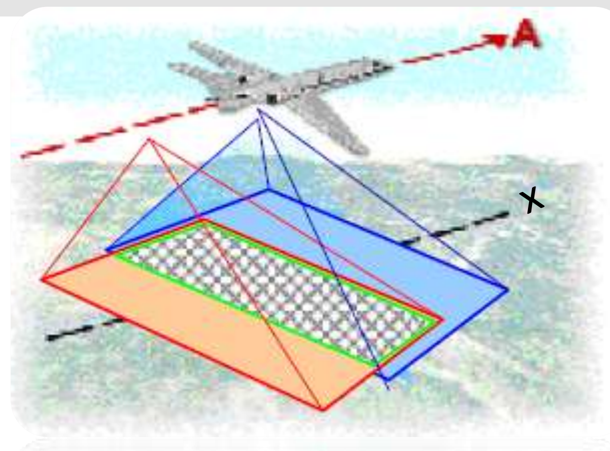
Definitions

Photo Coordinate System

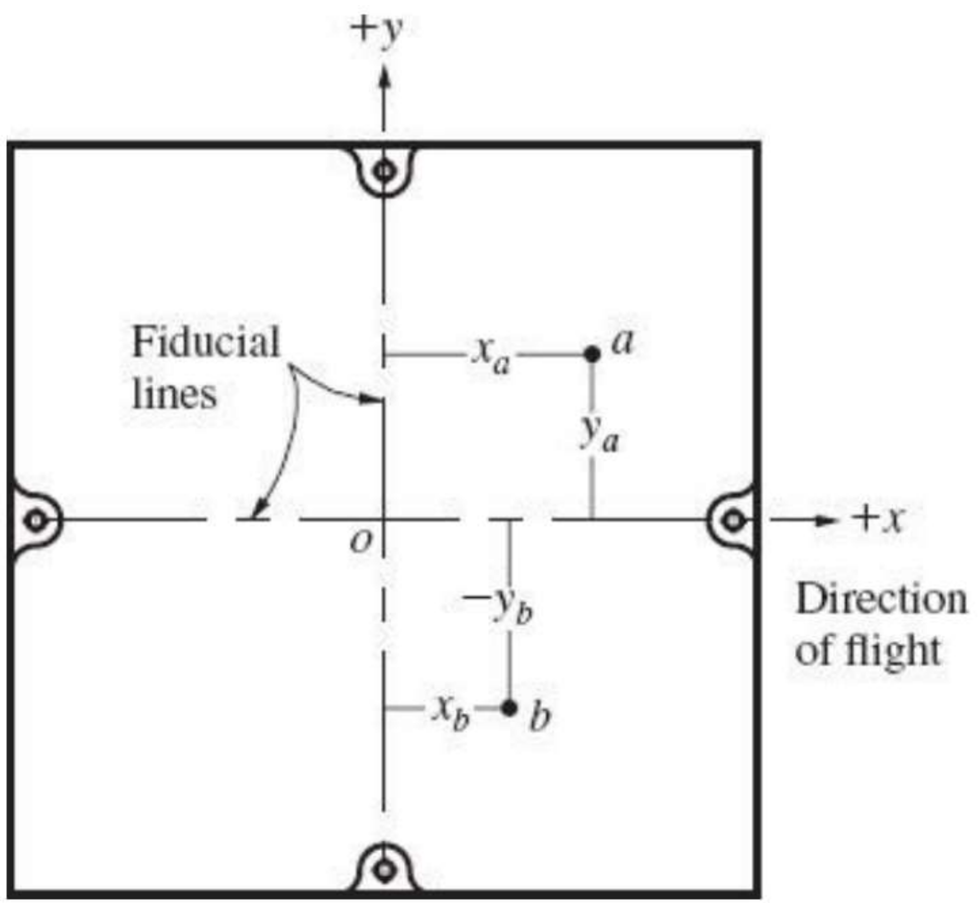
Defined by the fiducials in a film camera.
(x) axis is in direction of flight







By D. Kamal M. Ahmed



Photographic coordinate system based on side fiducials.

Photo Coordinates (film)

Repeated slide

- We use positives for ease of geometry and familiarity of feature shapes, negatives may be used in certain applications
- Lines connecting middle **fiducials ON THE POSITIVE** define a photo coordinate system, in which x is in the direction of flight, A RIGHT-HAND coordinate system

- Measurements can be as accurate as 1 micron = 1/1000 mm

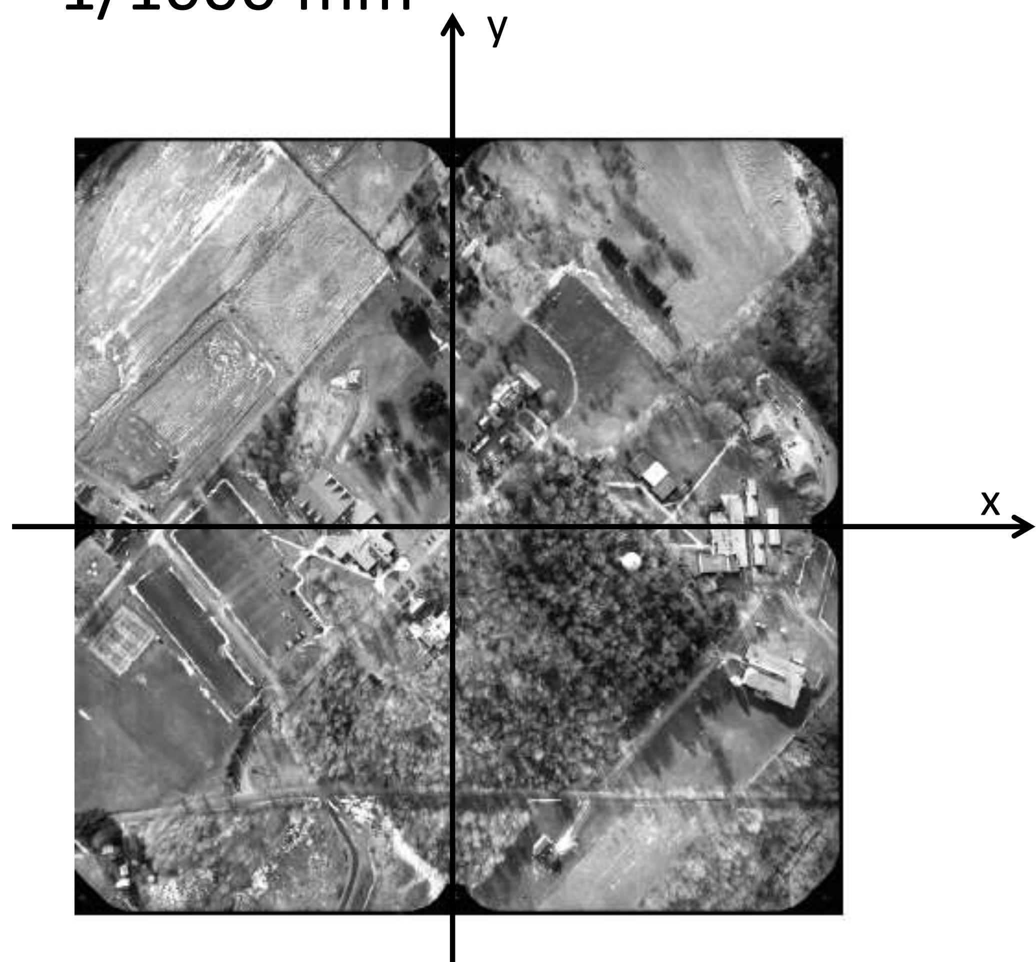
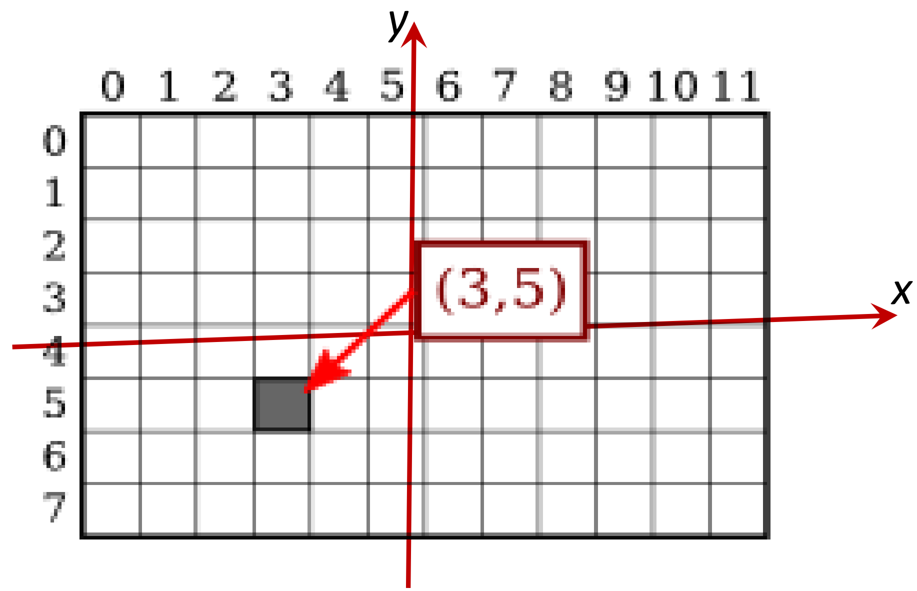
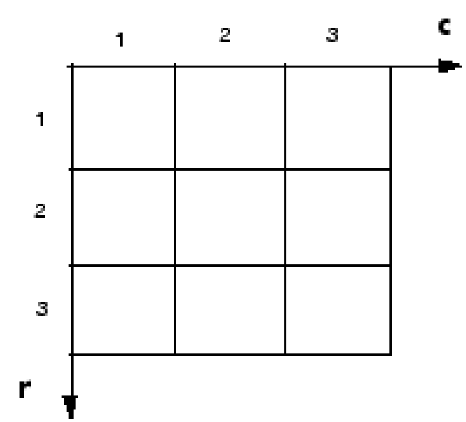


Photo Cooedinates (Digital) (x, y) and (r, c)

Repeated slide

- In a digital image, we measure rows and column locations(r, c).
- In a digital camera, the relationship between pixel locations in (r,c) and photo (x,y) at center is defined within the camera, no need for additional transformation
- In a film camera, the relationship is between photo fiducials (x,y) and ground (X, Y).
- When we scan a photo taken by a film camera, it becomes in a digital format, we measure (r,c). In this case, we need to transform (r,c) that we measure to (x,y) photo coordinates to apply the equations.
- A two-dimensional coordinate transformation from (r, c) to (x, y) is needed if the camera is not digital.





Geometry of aerial cameras

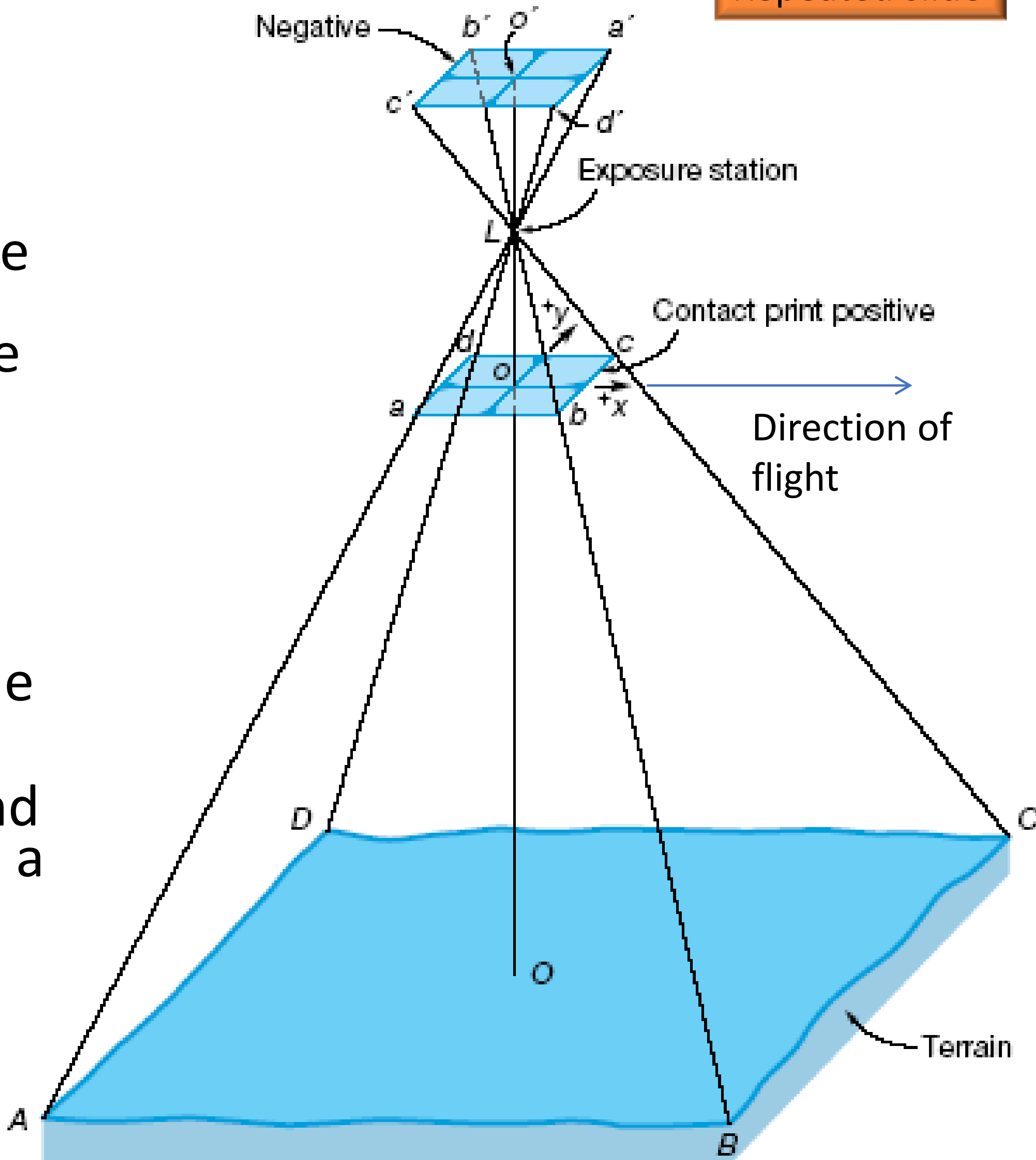
- Identify the following:

- Photo Coordinate System(right-handed)
- L: perspective center.
- Fiducial Center F.C.: intersection of (x), and (y) axis as defined by the fiducials (in a film camera).
- Principal Point (P.P) or O: the point where the perpendicular from the perspective center intersects the photograph. Usually deviates from the F.C by a very small distance.
- Principal axis: the line perpendicular from the principal center on the plane of the photograph (negative).
- f : the focal length, equals the Principal Distance.

Geometry of Vertical Photographs

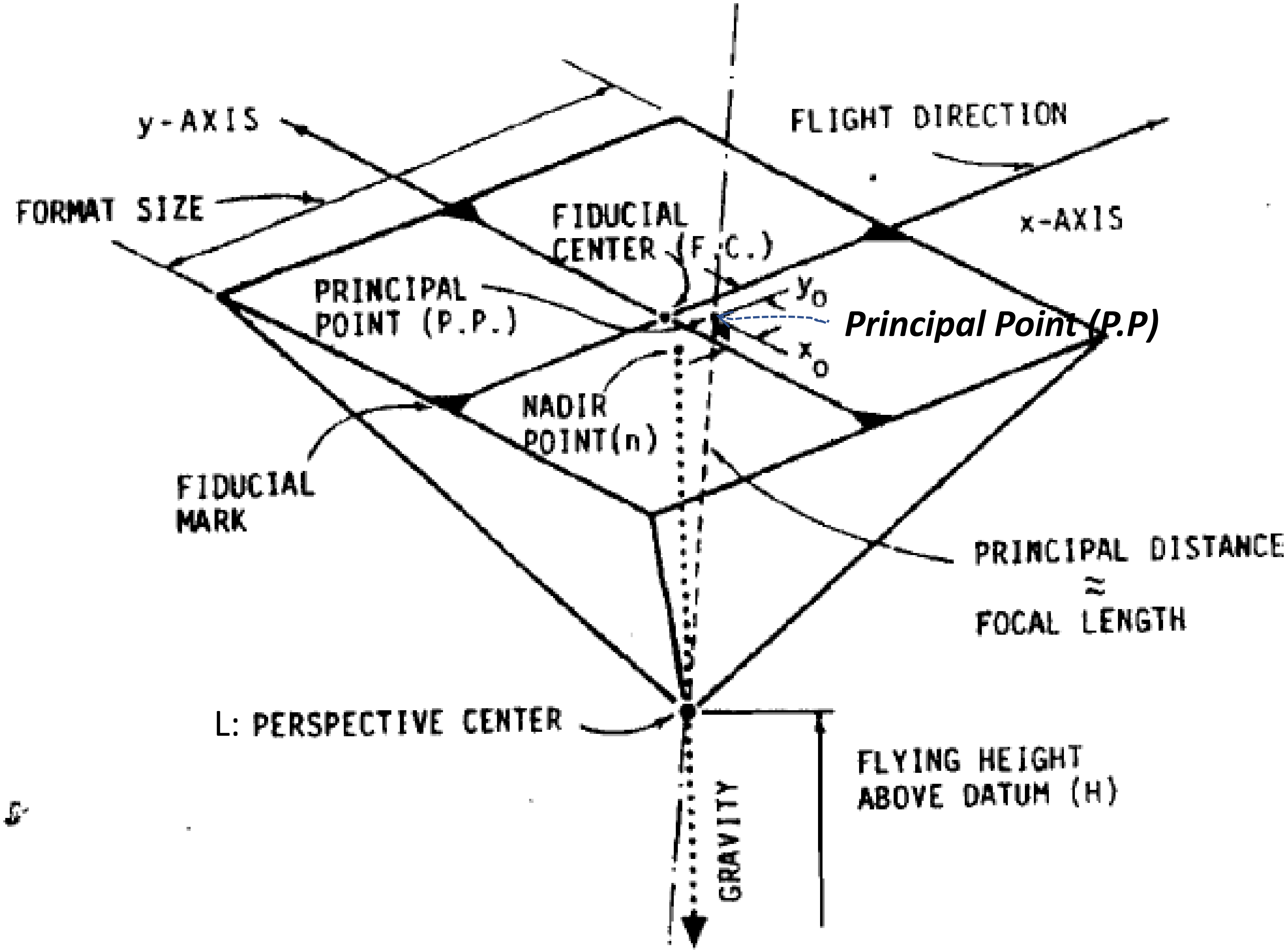
Repeated slide

- Define: image coordinate system (right handed), principal point, exposure station.
- If a film is used, measurements could be done using negatives or diapositives, same geometry, referred to the fiducials
- In digital images, row and columns of pixels define a coordinate system



Geometry of aerial cameras

Repeated slide



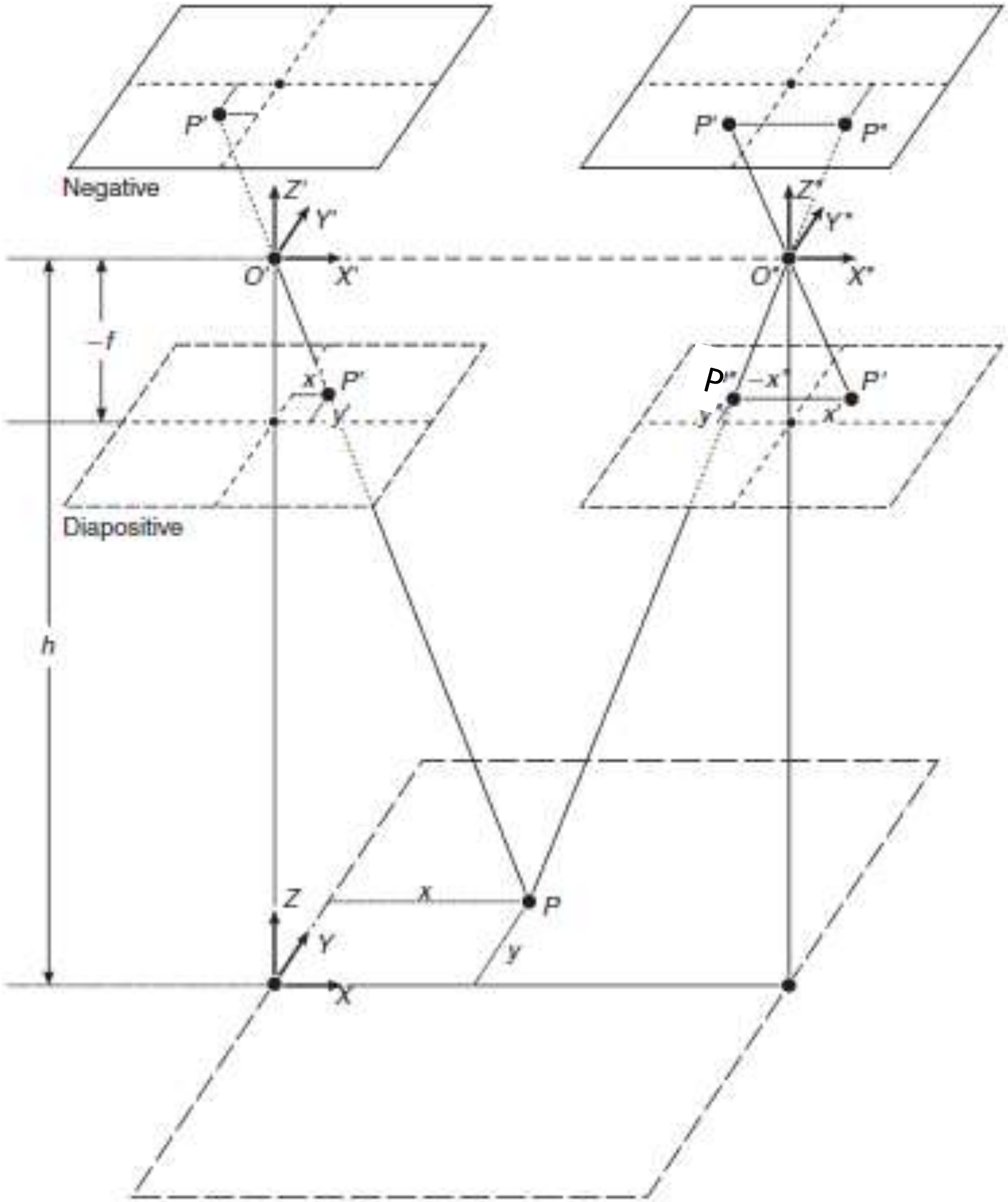
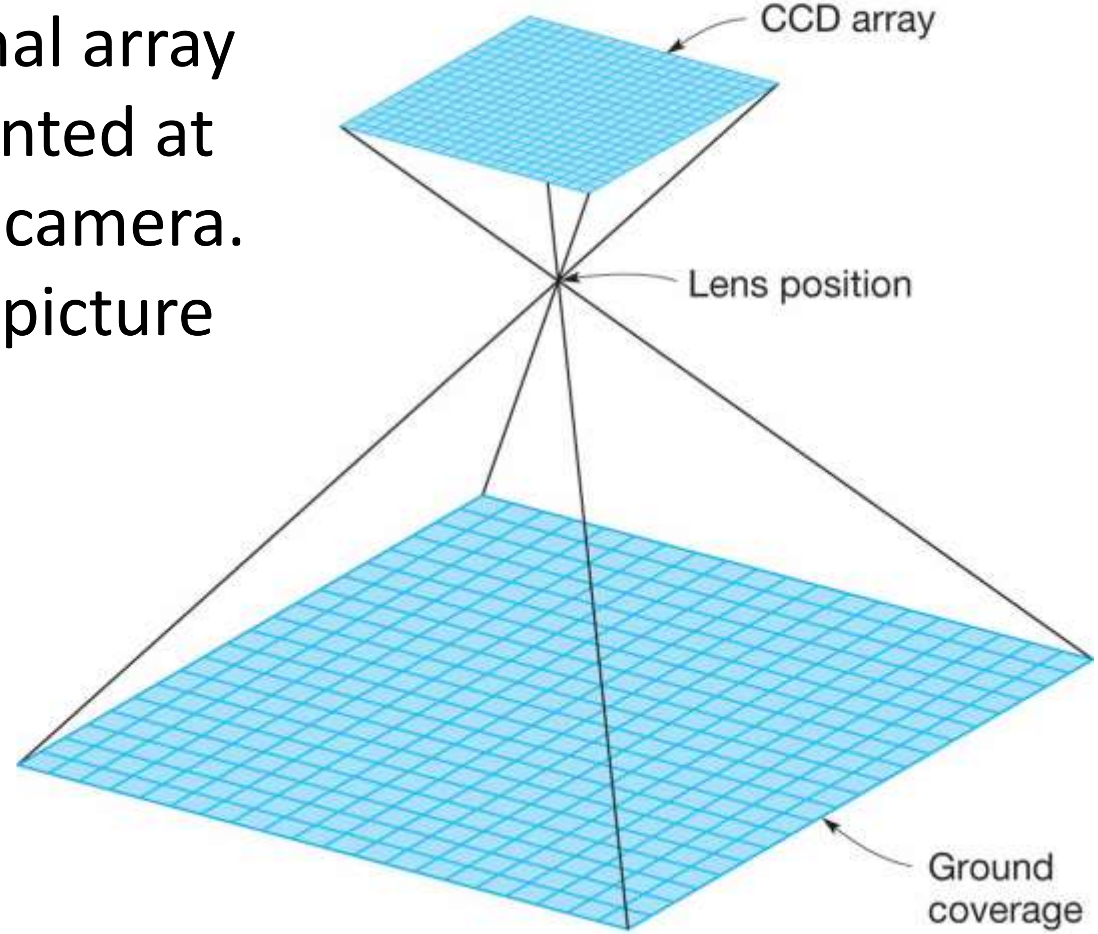


Figure 3.8 Geometry of vertical photographs.

Geometry of a digital frame camera

- Similar geometry is assumed in case of a digital camera
- Uses a two-dimensional array of CCD elements mounted at the focal plane of the camera.
- The image is a grid of picture elements (pixels)

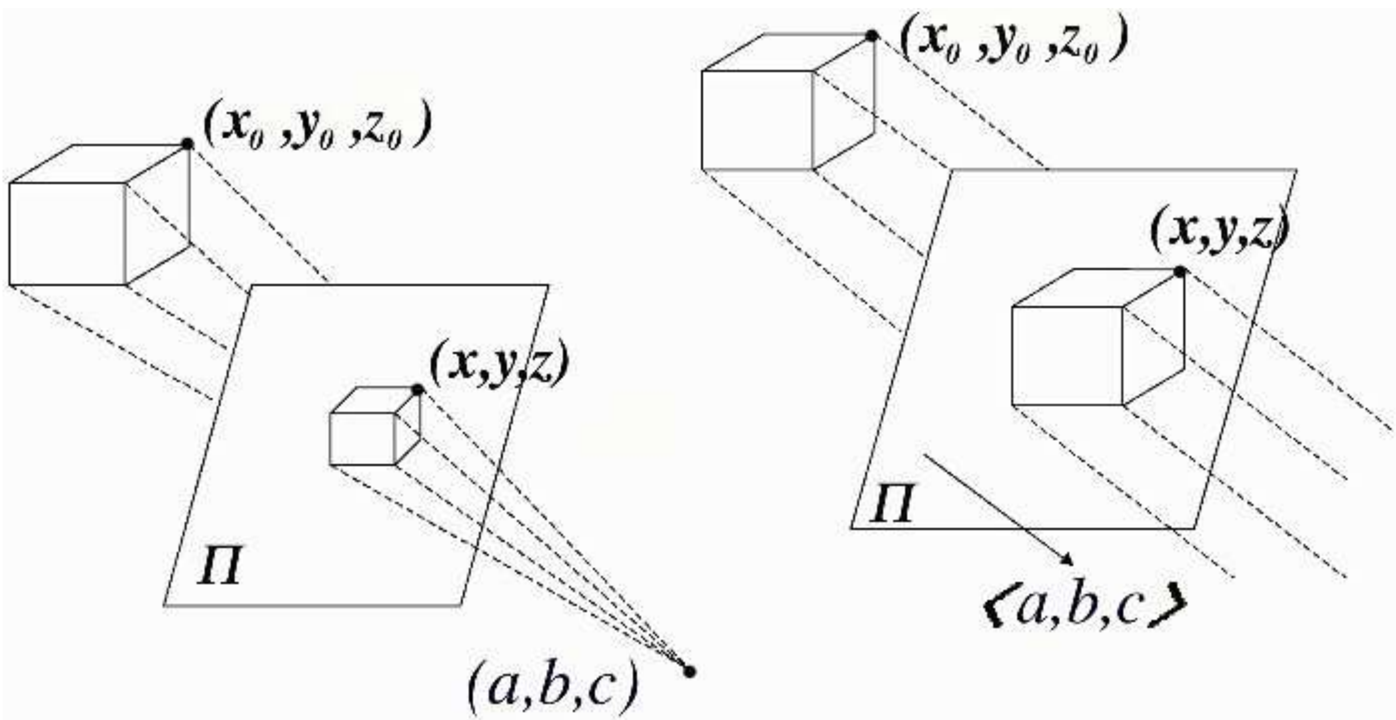


Vertical Photographs

Single Photo Applications
Scale

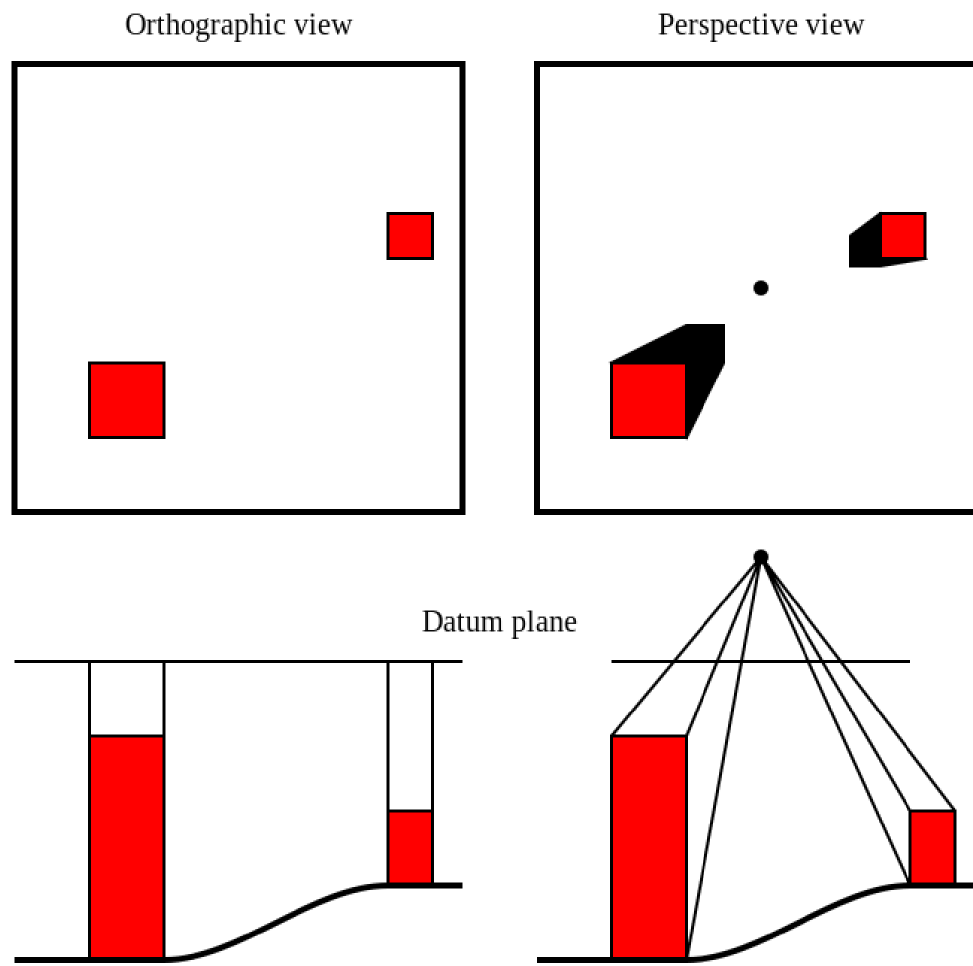
Scale of a Vertical Photograph

- Scale of a photograph is the ratio of a distance on a photo to the same distance on the ground.
- Differences between photographs and maps:
  - Projection: orthogonal or prospective
  - Scale constant on maps, but varies on photos. The closer the object to the camera (higher), the bigger it looks.



Orthophoto maps

- geometrically corrected images, can be used as maps.
- The shift in image is proportional to the relief " height" of location in a photo.
- A DTM is needed to correct "rectify" the image so that images are displaced to their corresponding location on maps.



Orthophoto from a drone

source: Wikipedia.com

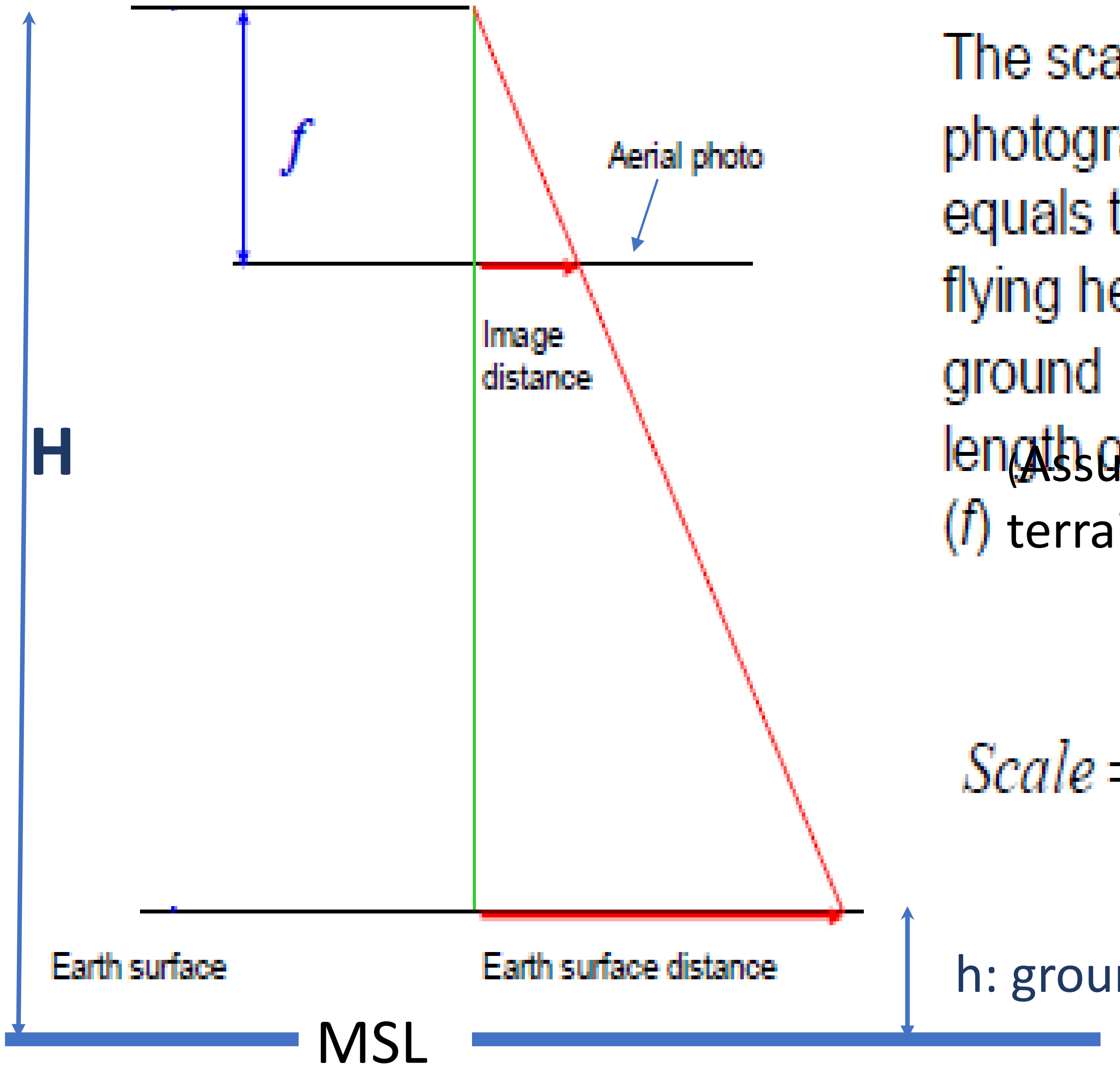
Orthophotos and True Orthophotos

- Orthophotos are corrected for relief of topography, top of tall structures are still shifted.
- True orthophotos are produced from multiple images to eliminate the displacement of the top of tall objects.

Orthophoto of Washington, DC

Orthophoto of the UW

- Scale of a photograph is the ratio of a distance on a photo to the same distance on the ground.

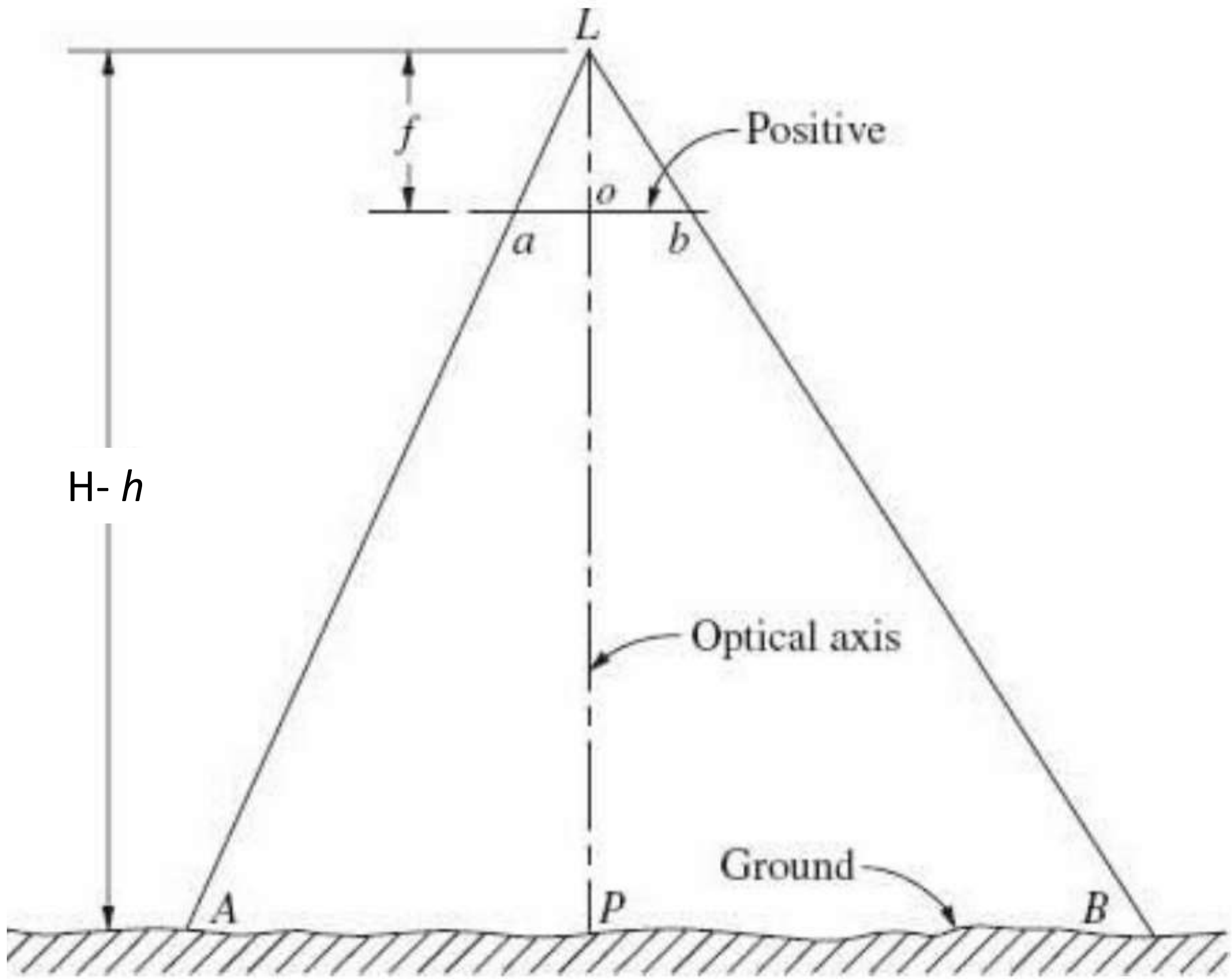


The scale of a vertical photograph approximately equals to the ratio of the flying height above the ground and the focal length of the camera lens (f)

(Assuming horizontal flat terrain)

$$Scale = \frac{imageDist}{surfaceDist} = \frac{f}{H-h}$$

h: ground elevation



- Scale of a vertical photo of flat horizontal ground is:

$$S = f/(H-h)$$

Aerial photograph taken at 460 m above ground, scale 1: 3,000

Aerial photograph taken at 910 m above ground, scale 1: 6,000

Aerial photograph taken at 1830 m above ground, scale 1: 12,000

Note that all the photos are of the same area.

Aerial photograph taken at 3660 m above ground, scale 1: 24,000

Example

A vertical aerial جويهphotograph is taken over flat terrain ارض with a 152.4 mm-focal-length camera from an altitude ارتفاع of 1830 m above ground. What is the photo scale?

- Answer:

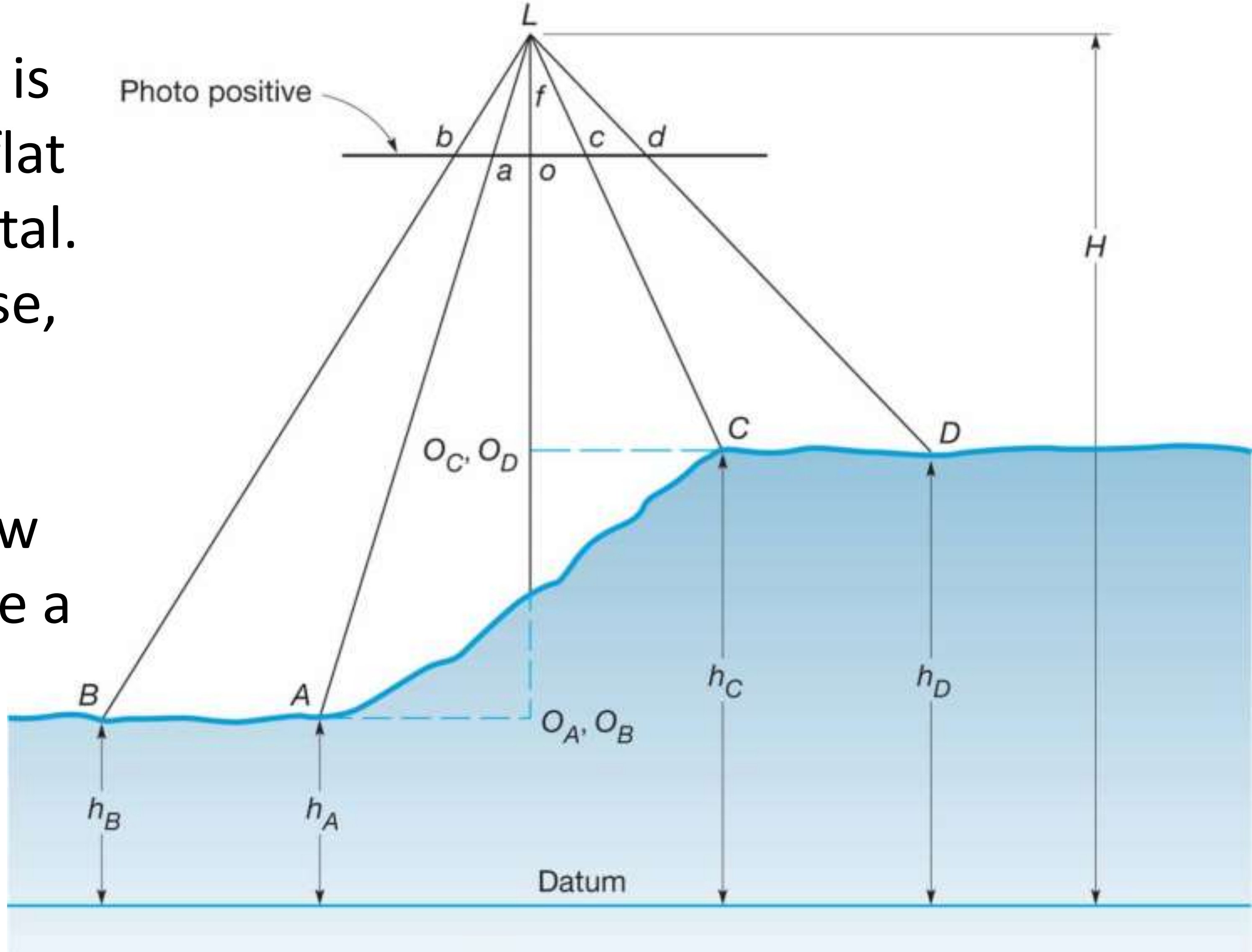
S = 152.4 mm/ 1830 m

= 0.1524 m / 1830 m =

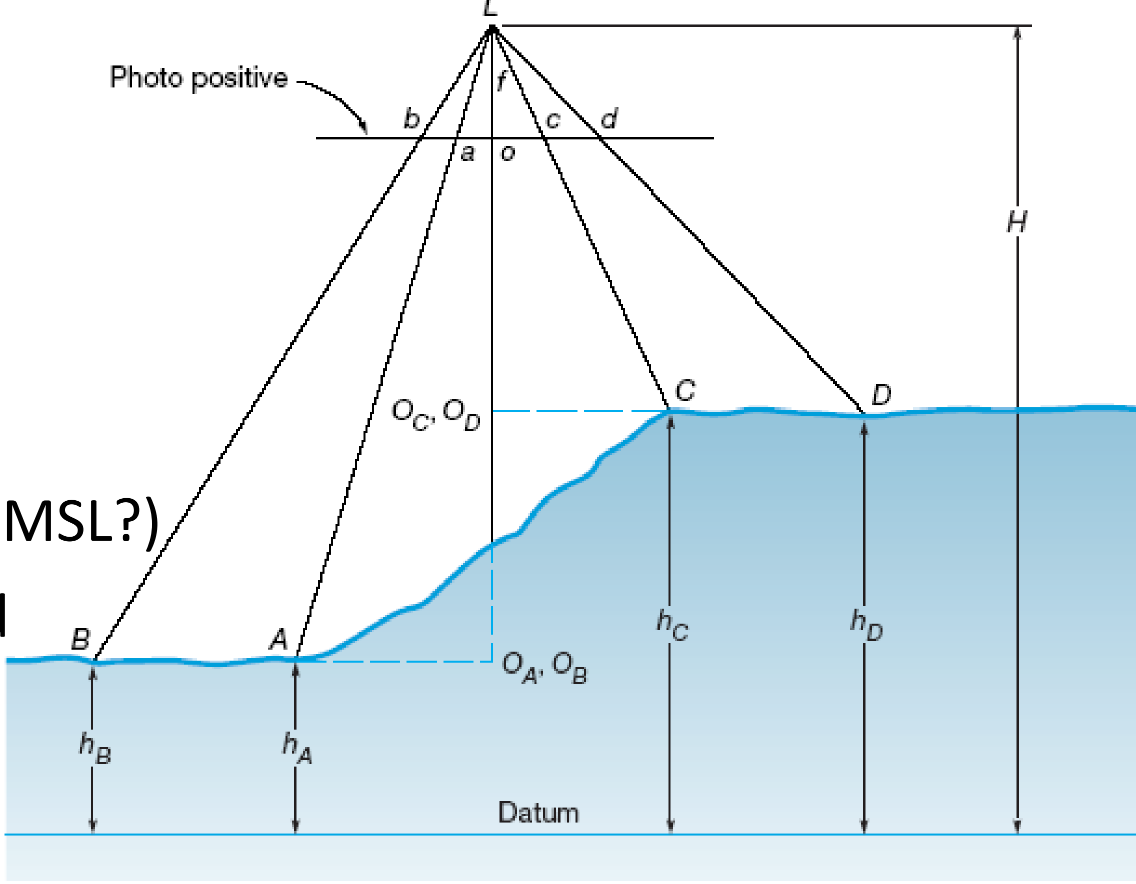
1: 12,000

Scale of a vertical photograph over variable terrain

But, ground is not always flat and horizontal. H in that case, elevation of ground, is variable, how do we define a scale??



Scale of a Vertical Photograph



- Definitions:

f: focal length of the camera

H: flying hight above datum (MSL?)

h: flying height above ground

- Scale (s) at any point:

$$S = \frac{f}{H - h}$$

- Average scale of a photograph:

$$S_{avg} = \frac{f}{H - h_{avg}}$$

- If f, H, and h are not available, but a map is available then:

$$\text{Photo Scale} = \frac{\text{photo distance}}{\text{map distance}} \text{ X map scale}$$

Example

Suppose that highest terrain h_1, average terrain h_{avg}, and lowest terrain h_2 of Fig. 6-3 are 610, 460, and 310 m above mean sea level, respectively. Calculate the maximum scale, minimum scale, and average scale if the flying height above mean sea level is 3000 m and the camera focal length is 152.4 mm.

Answer:

Example

On a vertical photograph the length of an airport runway measures 160 mm. On a map that is plotted at a scale of 1:24,000, the runway is measured as 103 mm. What is the scale of the photograph at runway elevation?

Answer

Vertical Photographs

Single Photo Applications
Local Ground Coordinates

Ground Coordinates from a Single Vertical Photograph

- With a local image coordinate system defined, we define an arbitrary ground coordinate system.
- That ground system could be used to compute distances and azimuths. Coordinates can also be transformed to any system
- In that ground system:

$$X_a = x_a * \text{(photograph scale at a)}$$

$$Y_a = y_a * \text{(photograph scale at a)}$$

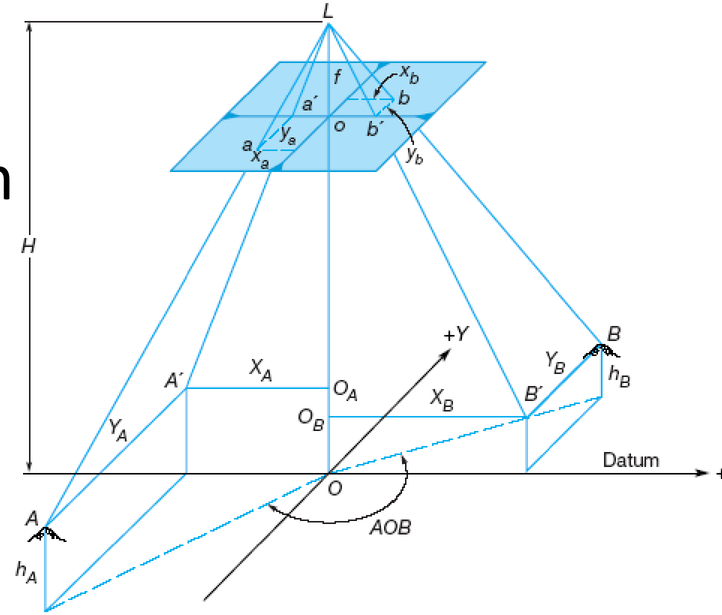


Figure 27–8 Ground coordinates from a vertical photograph.

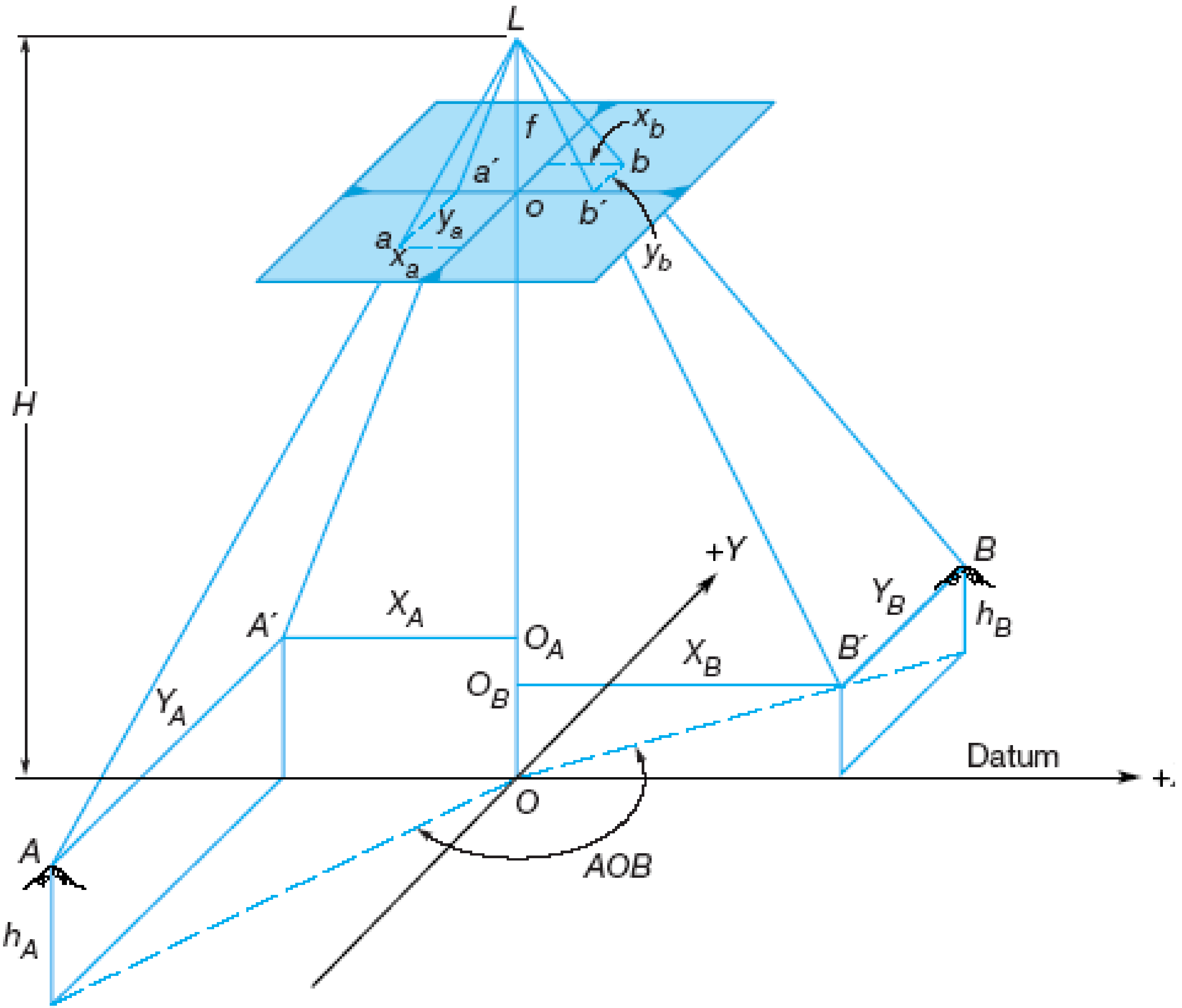


Figure 27–8 Ground coordinates from a vertical photograph.

Example (consider answering on your own)

A vertical photograph was taken with a camera having a focal length of 152.3 mm. Ground points A and B have elevations 437.4 m and 445.3 m above sea level, respectively, and the horizontal length of line AB is 584.9 m. The images of A and B appear at a and b, and their measured photo coordinates are x_a = 18.21 mm, y_a = –61.32 mm, x_b = 109.65 mm, and y_b = –21.21 mm. Calculate the flying height of the photograph above sea level.

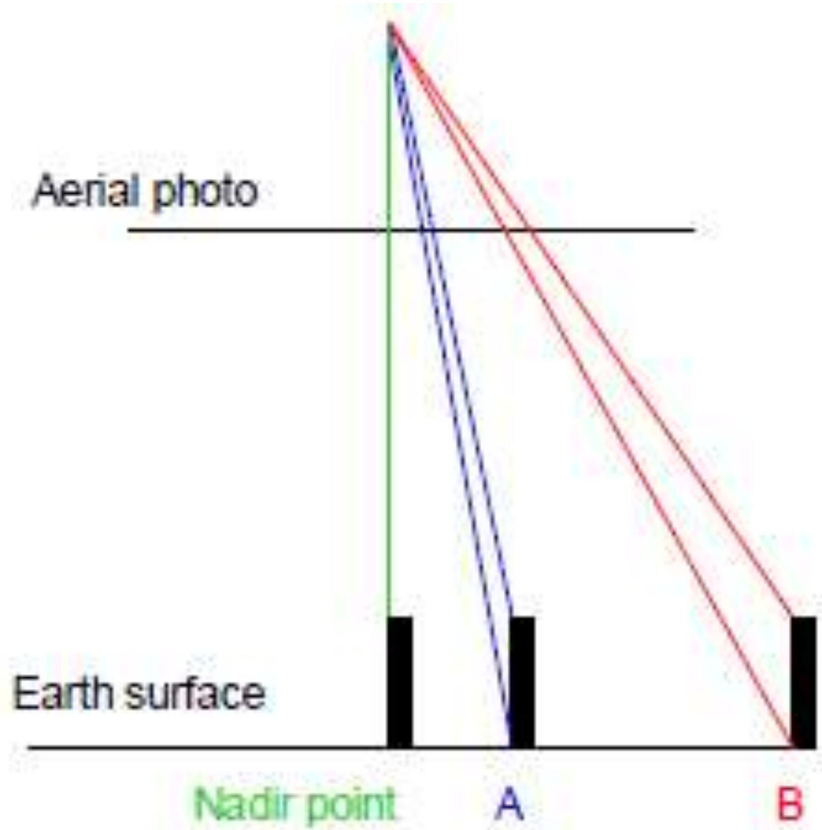
Answer:

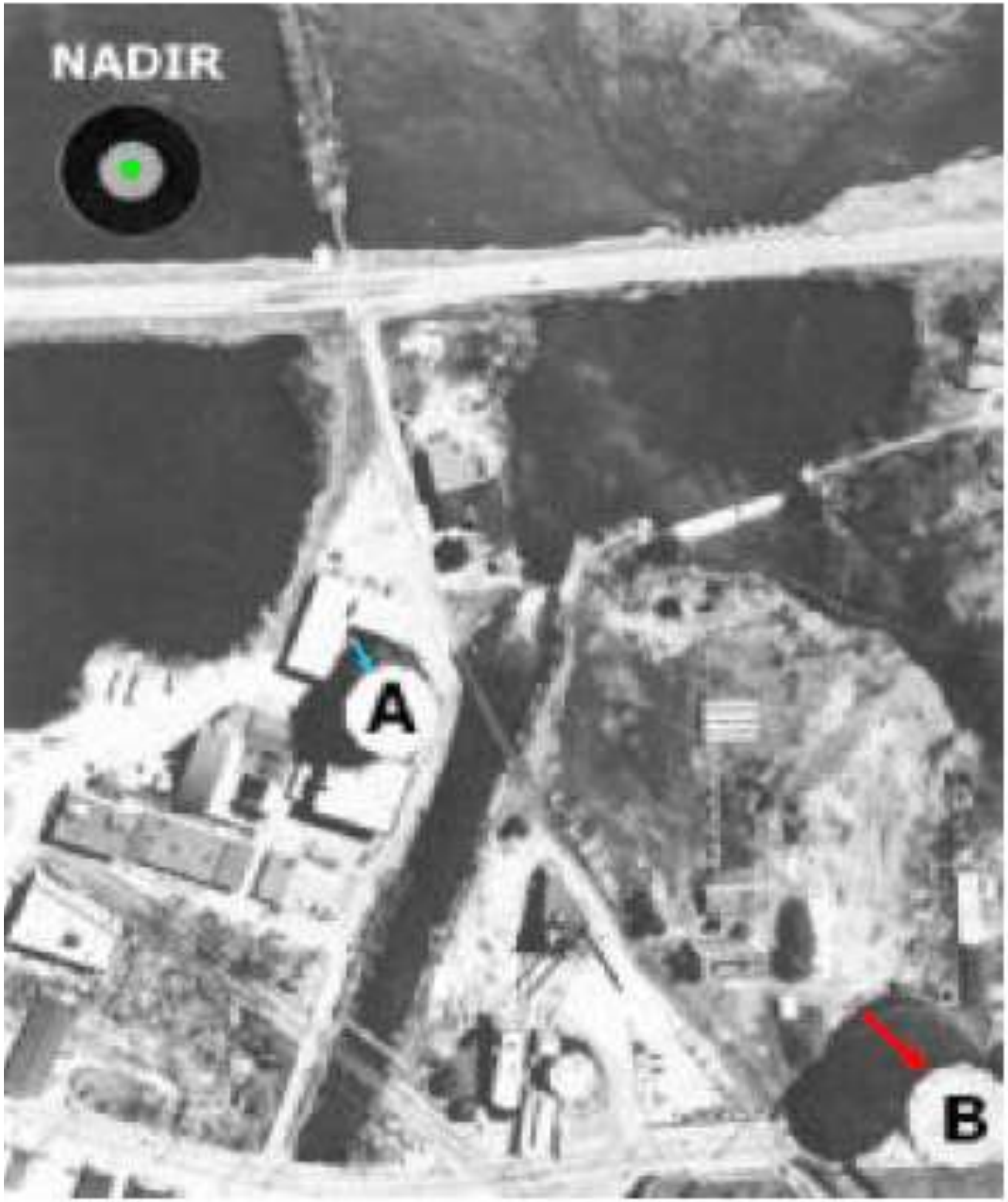
Vertical Photographs

Single Photo Applications
Relief displacement

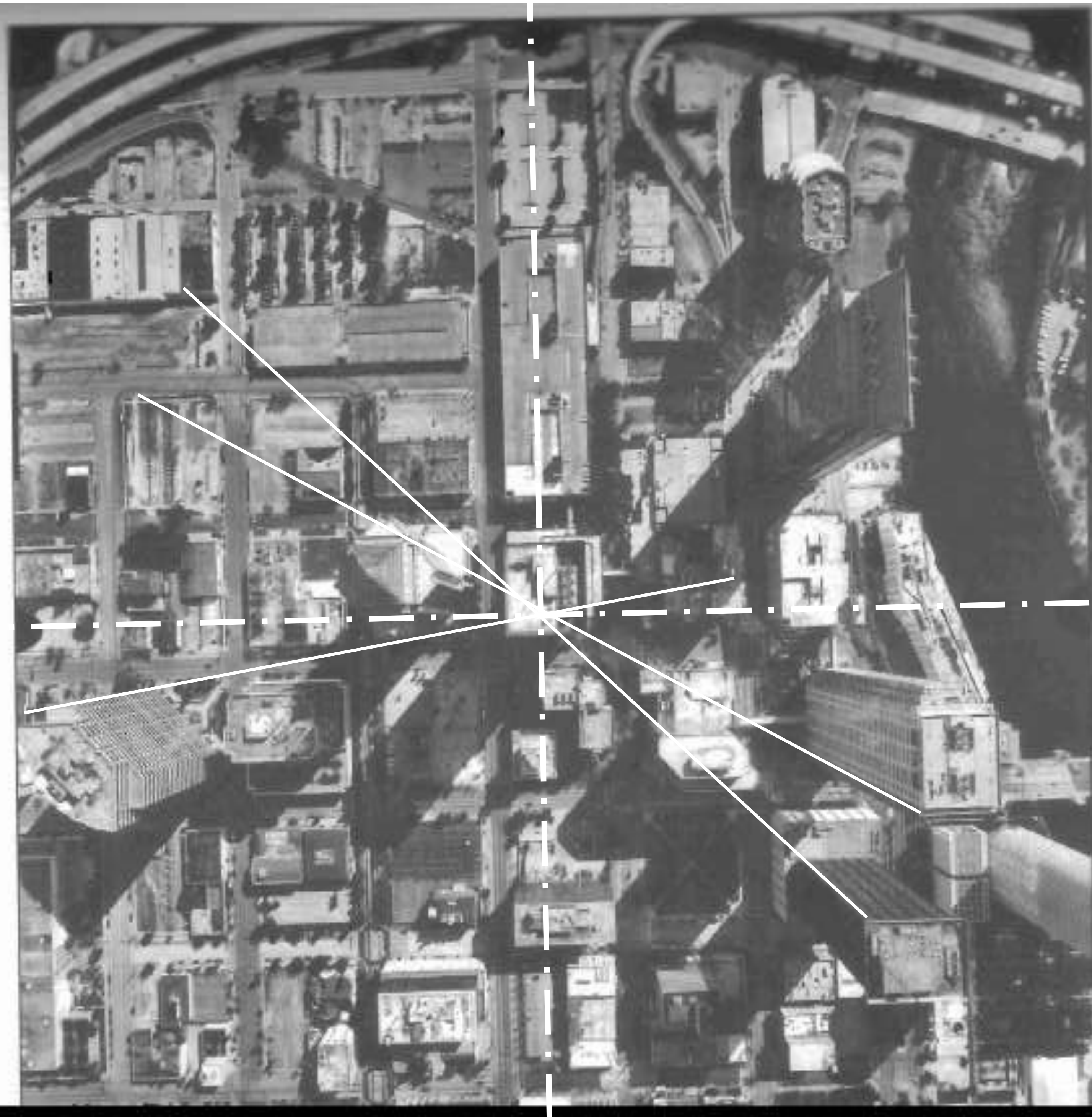
Relief displacement

Towers A and B are equally high, but placed at different distances from the nadir point, thus have different relief displacements. A tower, depicted beneath nadir point has no relief displacement





Relief displacement from Nadir (enlarged)



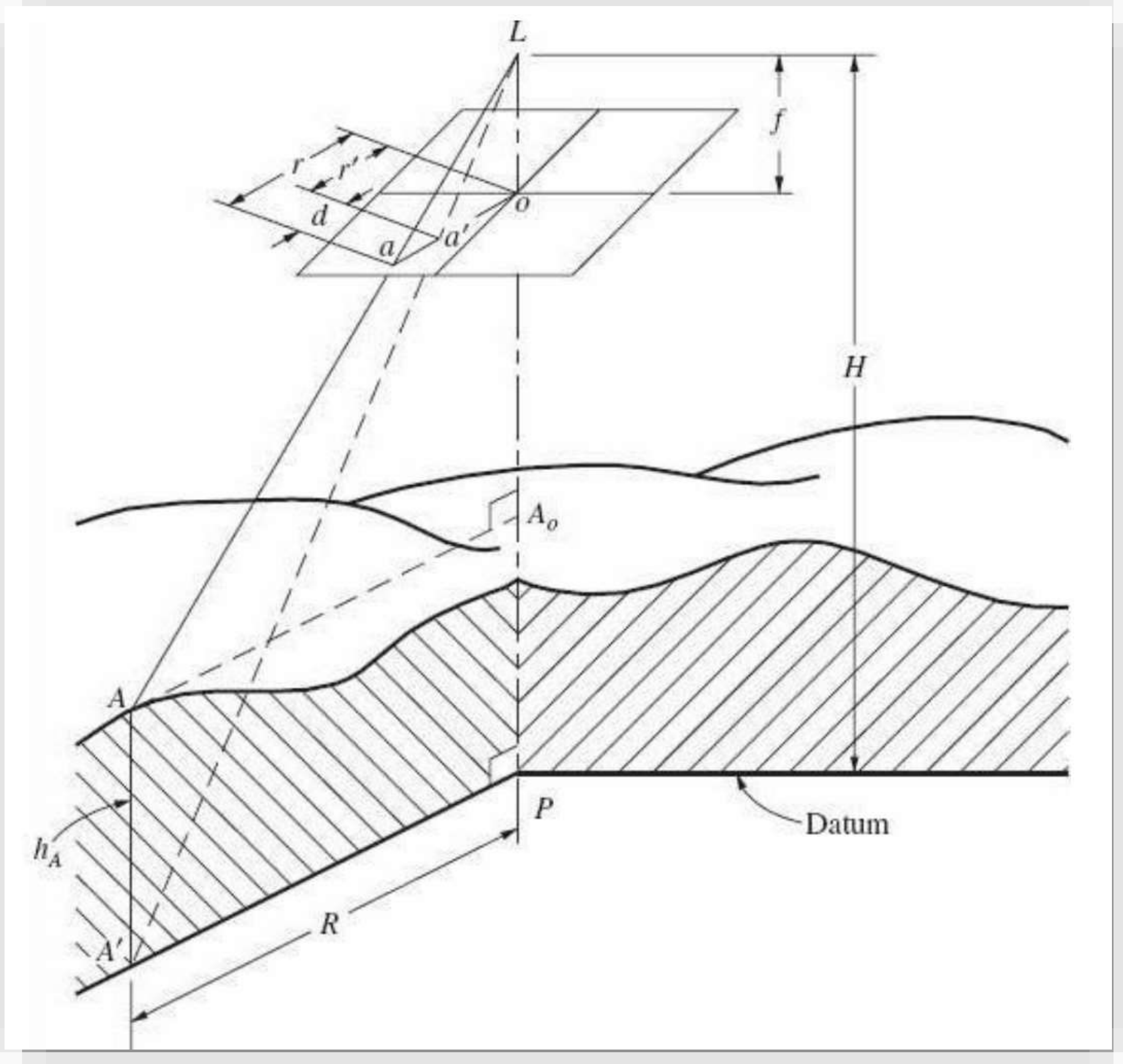
Relief displacement from Nadir (Center)

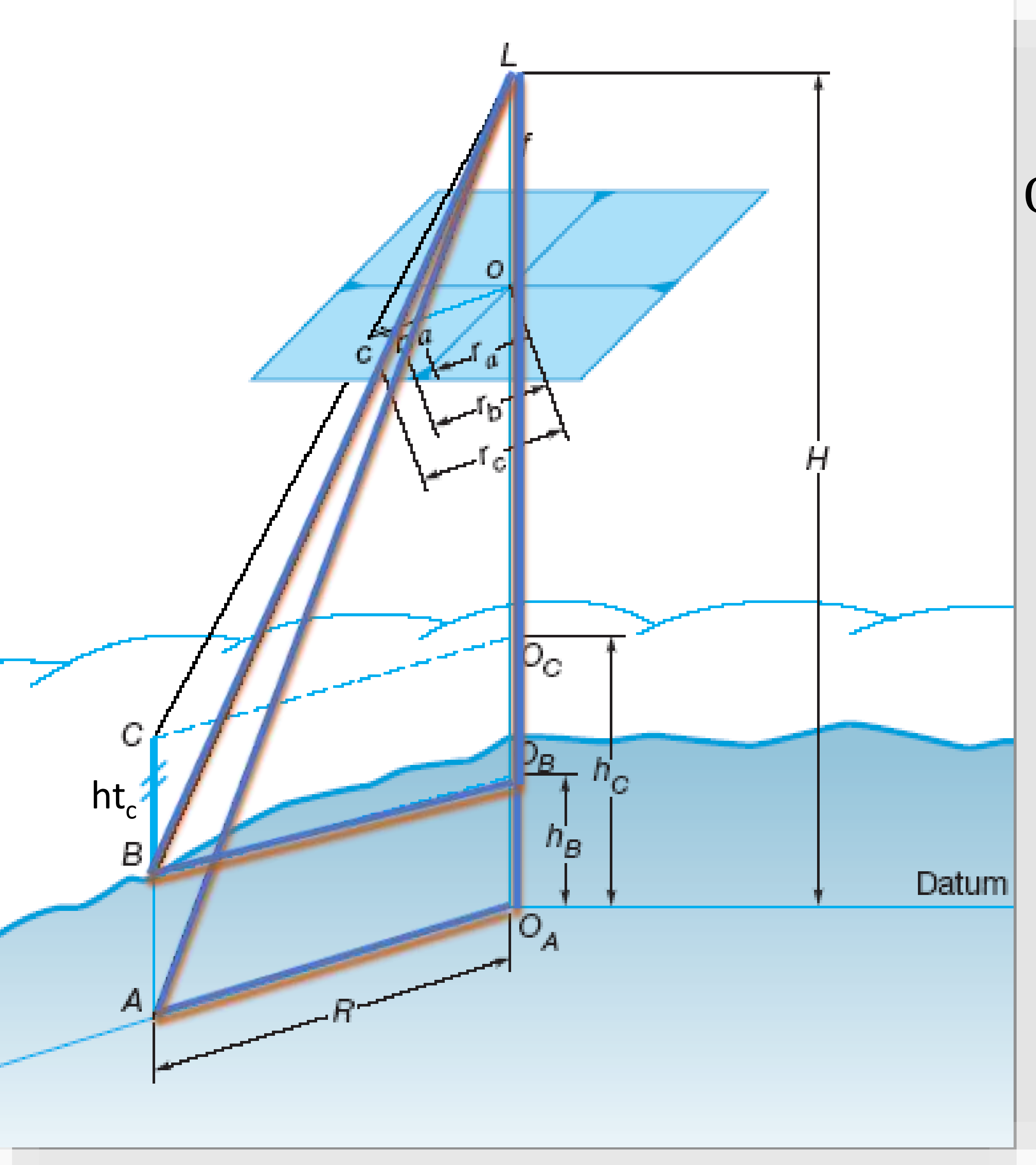
Relief Displacement on a Vertical Photograph الازاحه بسبب الارتفاع

The shift of an image from its location as caused by the object's relief. Two points on a vertical line will appear as one line on a map, but two points, usually, on a photograph. الازاحه لنقطه علي الصوره من مكانها بسبب الارتفاع

- In a vertical photo, the displacement is from the principal point.

Example of geometry





$r_a/R = f/H$

Or: $r_a * H = R * f$ ----(1)

Similarly:

$r_b/R = f/(H-h)$

=> $r_b * (H-h) = R * f$ ---(2)

Then from (1) and (2);

$r_a * H = r_b * (H-h)$ then;

$(r_b * H) - (r_a * H) = r_b h$

$d_b = r_b - r_a = r_b * h_b / H$

- Relief displacement (d) of a point with respect to a point on the datum :

$$d = \frac{r\,h}{H}$$

where:

r: is the radial distance on the photo to the high point

h : elevation of the high point, and H is flying height above datum

- Assuming that the datum is at the bottom of vertical object, H is the flying height above ground, the value h will compute the object height.

Also:

$r_c/R = f/\{(H\text{-}h) - ht_c\}$

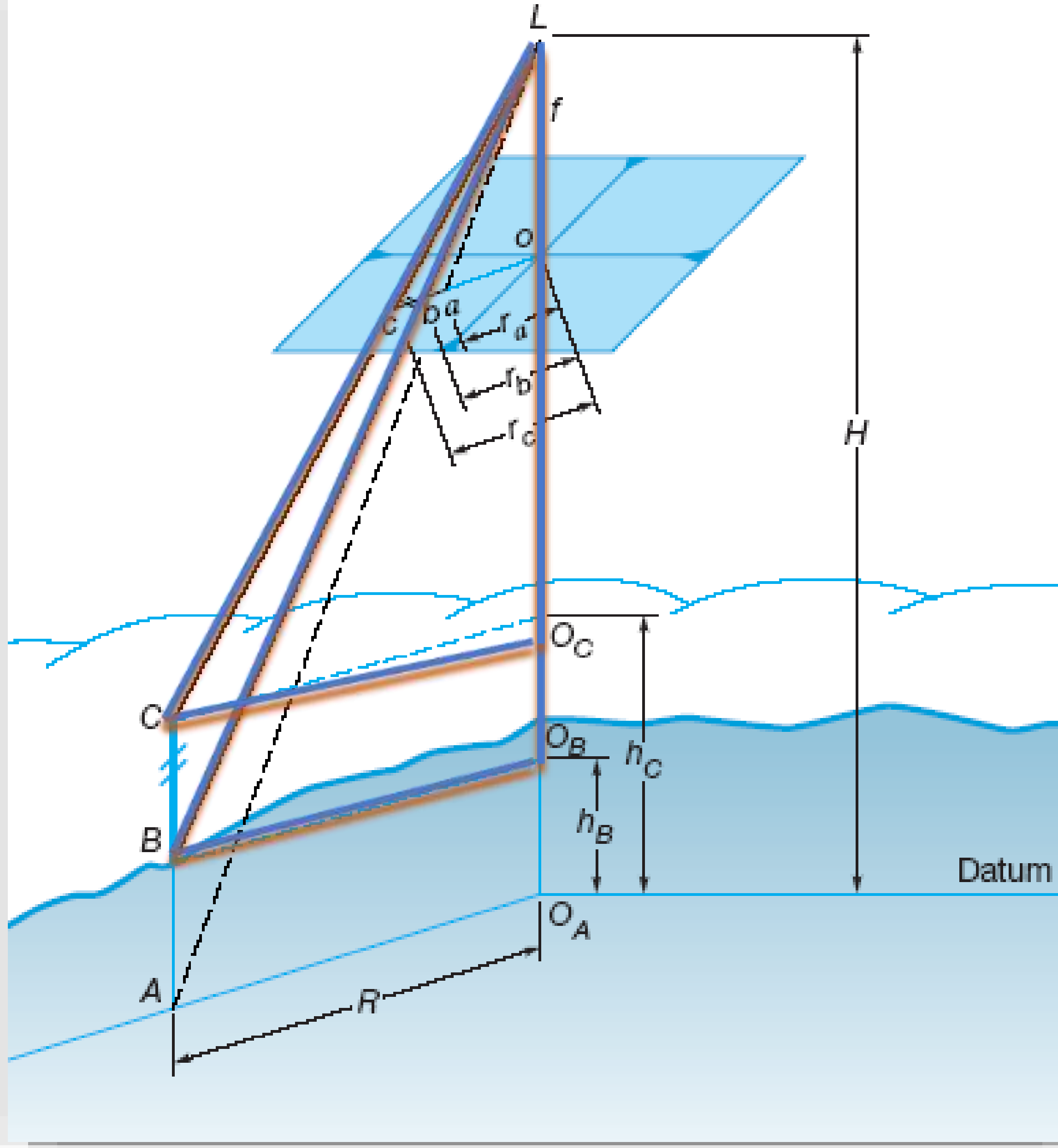
$\Rightarrow r_c * \{(H\text{-}h) - ht_c\} = R*f$ ---(3)

From (2) and (3):

$r_c * (H\text{-}h) - (r_c * ht_c) = r_b * (H\text{-}h)$ then;

$(r_c - r_b)*(H\text{-}h) = r_c\, ht_c$

$d = r_c - r_b = r_c * ht_c / (H\text{-}h)$



In general:

Assume that point C is vertically above B, they are shown on the photograph as (c) and (b).
Measured radial distances from the center to points c and b (r_c and r_b) , then

$d_c = r_c - r_b$ and;
$d_c = (r_c * ht_c) / ($flying height above ground $= H – h_b)$

Note that relief displacement is eliminated in true ortho photos

Example:

A vertical photograph taken from an elevation of 535 m above mean sea level (MSL) contains an image of a tall vertical radio tower. The elevation at the base of the tower is 259 m above MSL. The relief displacement d of the tower was measured as 54.1 mm. What is the height of the tower?

Answer:

Vertical Photographs

Single Photo Applications
Flying Height

Flying Height of a Vertical Photograph

- Flying height can be determined by:
  - Readings on the photos
  - Applying scale equation, if scale can be computed
    - Example: what is the flying height above datum if f=6”, average elevation of ground is 900ft, scale is 1”:100ft? Is it 1500’?
  - Or, if two control points appear in the photograph, solve the equation:

$$L^2 = (X_B - X_A)2 + (Y_B - Y_A)^2$$

then solve the same equation again replacing the ground coordinates with the photo coordinates. Get the scale.

Ground Coordinates from a Single Vertical Photograph

- With image coordinate system defined, we may define an arbitrary ground coordinate system parallel to (x,y) origin at nadir.
- That ground system could be used to compute distances and azimuths. Coordinates can also be transformed to any system
- In that ground system:

$$X_a = x_a * \text{(photograph scale at a)}$$
$$Y_a = y_a * \text{(photograph scale at a)}$$

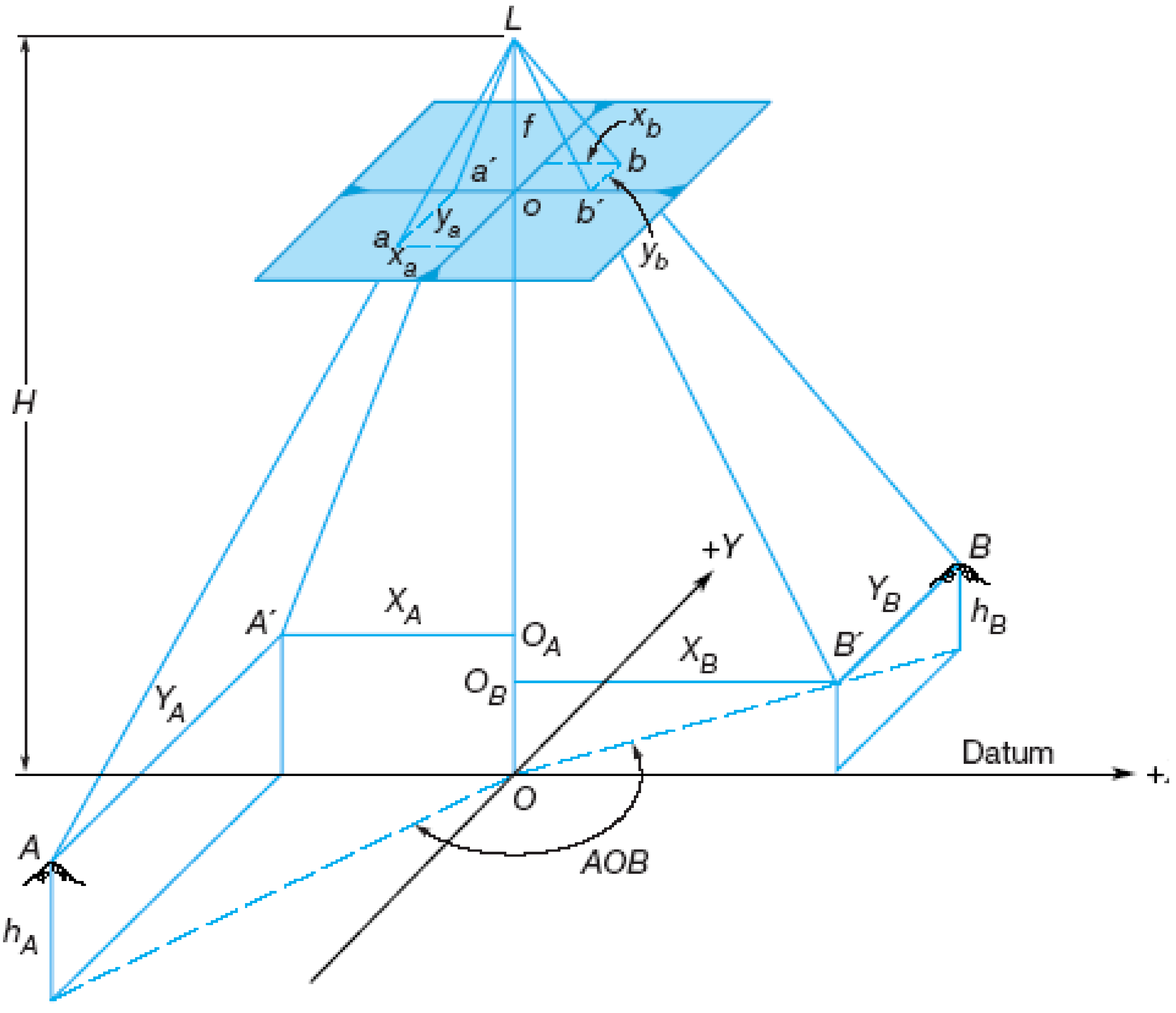
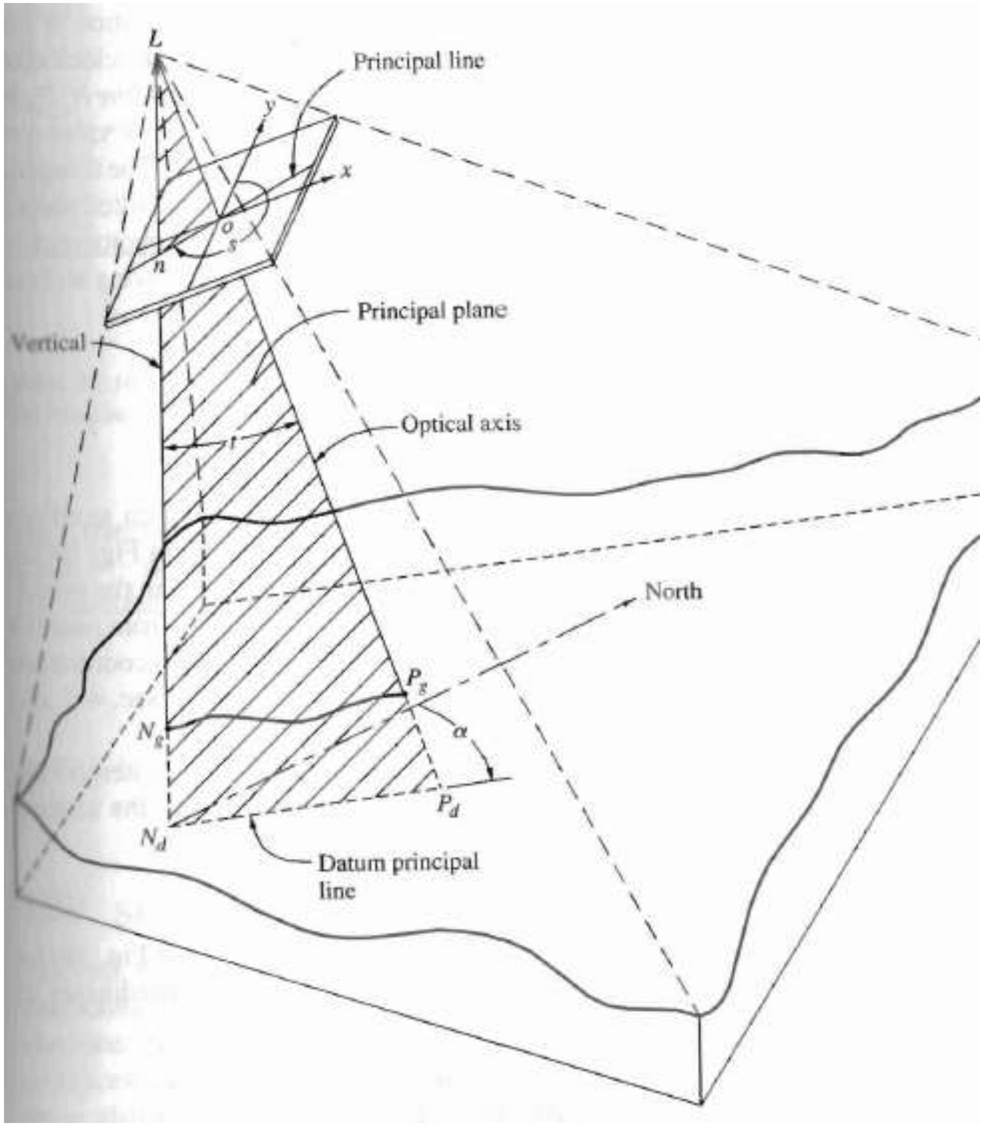


Figure 27–8 Ground coordinates from a vertical photograph.

Tilted Photographs

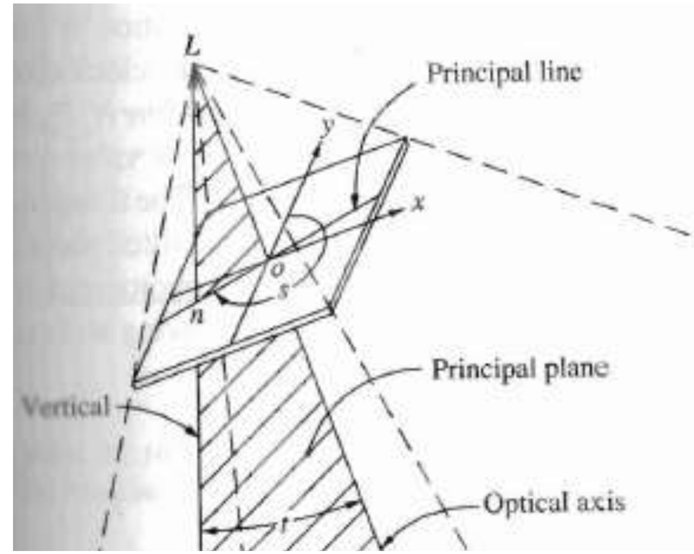
Tilted Photographs



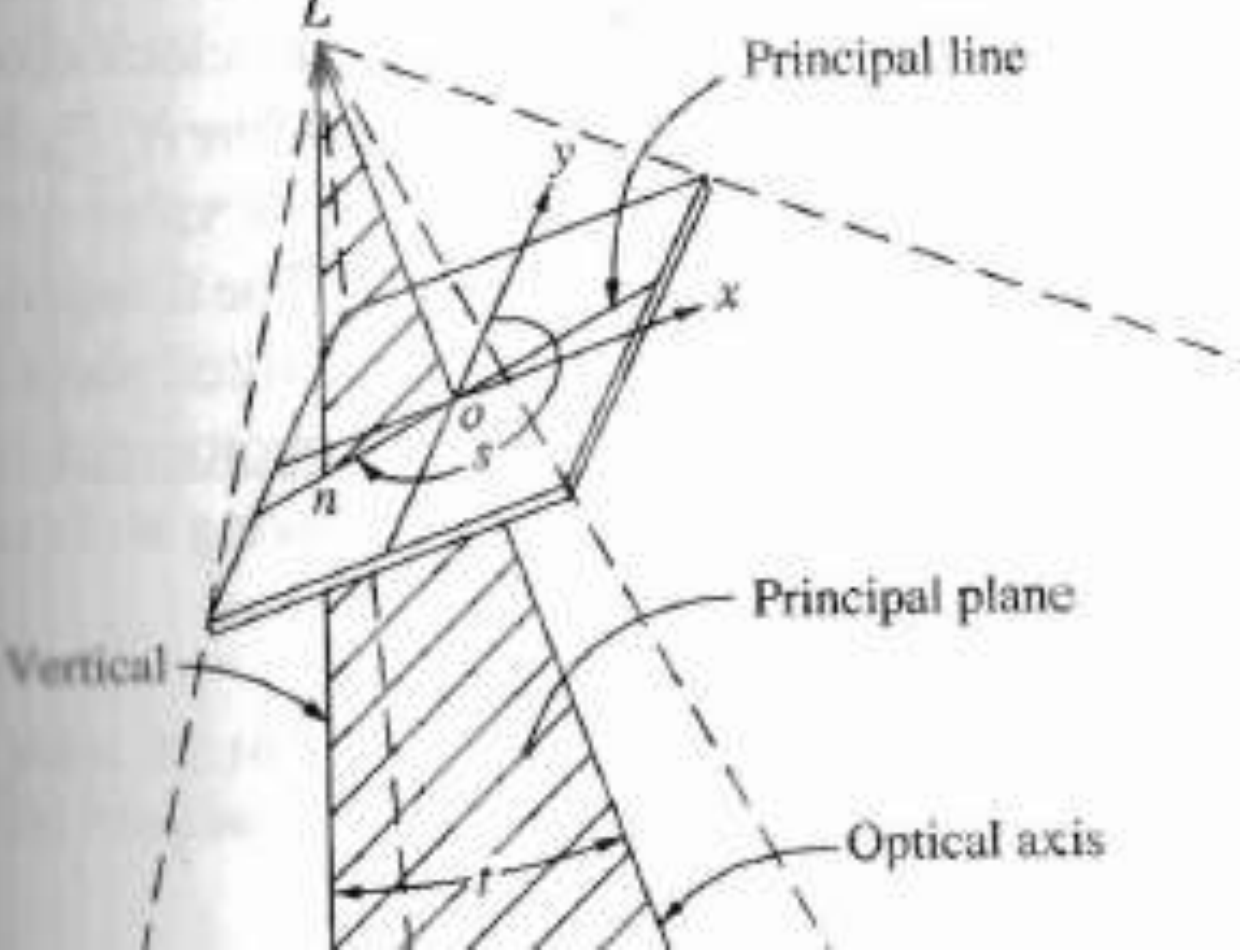
Basic elements of a tilted photographs

- The optical axis is tilted from the vertical
- Identify the following:

- t = angle of tilt between the plumb line and the optical axis L0



- i = the isocenter: the line bisecting the tilt angle intersects the principal line in the isocenter.
- no = the principal line joining the nadir point (n) and the principal point (0).



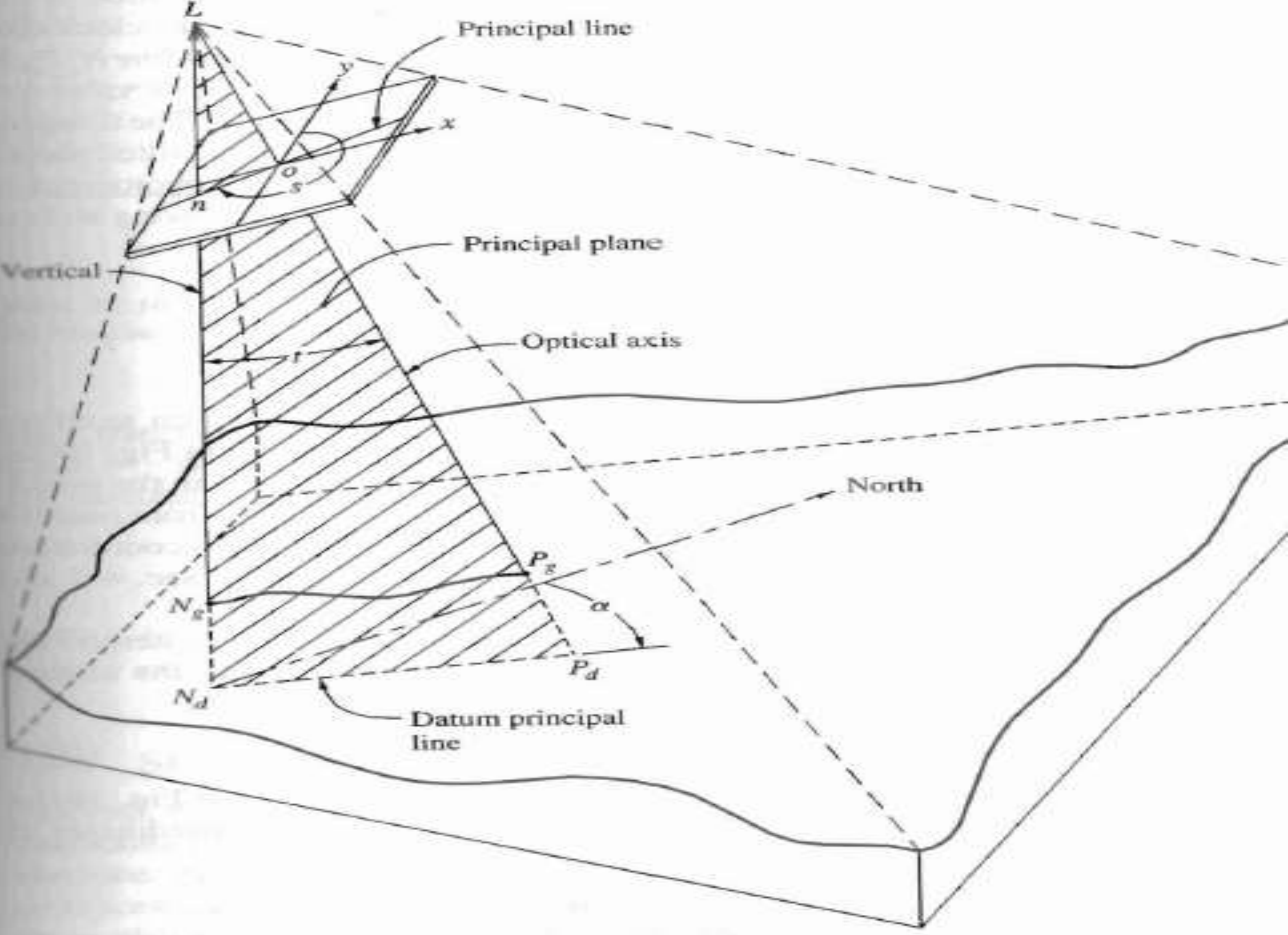
L
Principal line
y
x
o
s
n
Principal plane
Vertical
Optical axis
t

- Lno = the principal plane: it is the vertical plane containing o, L and n (shaped plane).
- *im* = axis of tilt: it is the line perpendicular to the principal line from the isocenter *i* in the plane of the photograph.
- S = the swing angle: it is the angle measured from the positive photographic y-axis clockwise to the principal line (on).
- x'y' axes are the auxiliary coordinate system of the tilted photograph where:

y' is the principal line (no).

x' is the perpendicular to y' from point *n* .

θ = the rotation angle between y and y' axes in a counterclockwise direction.


L
Principal line
y
x
o
s
n
Principal plane
Vertical
Optical axis
t
North
P_g
N_g
α
P_d
N_d
Datum principal
line

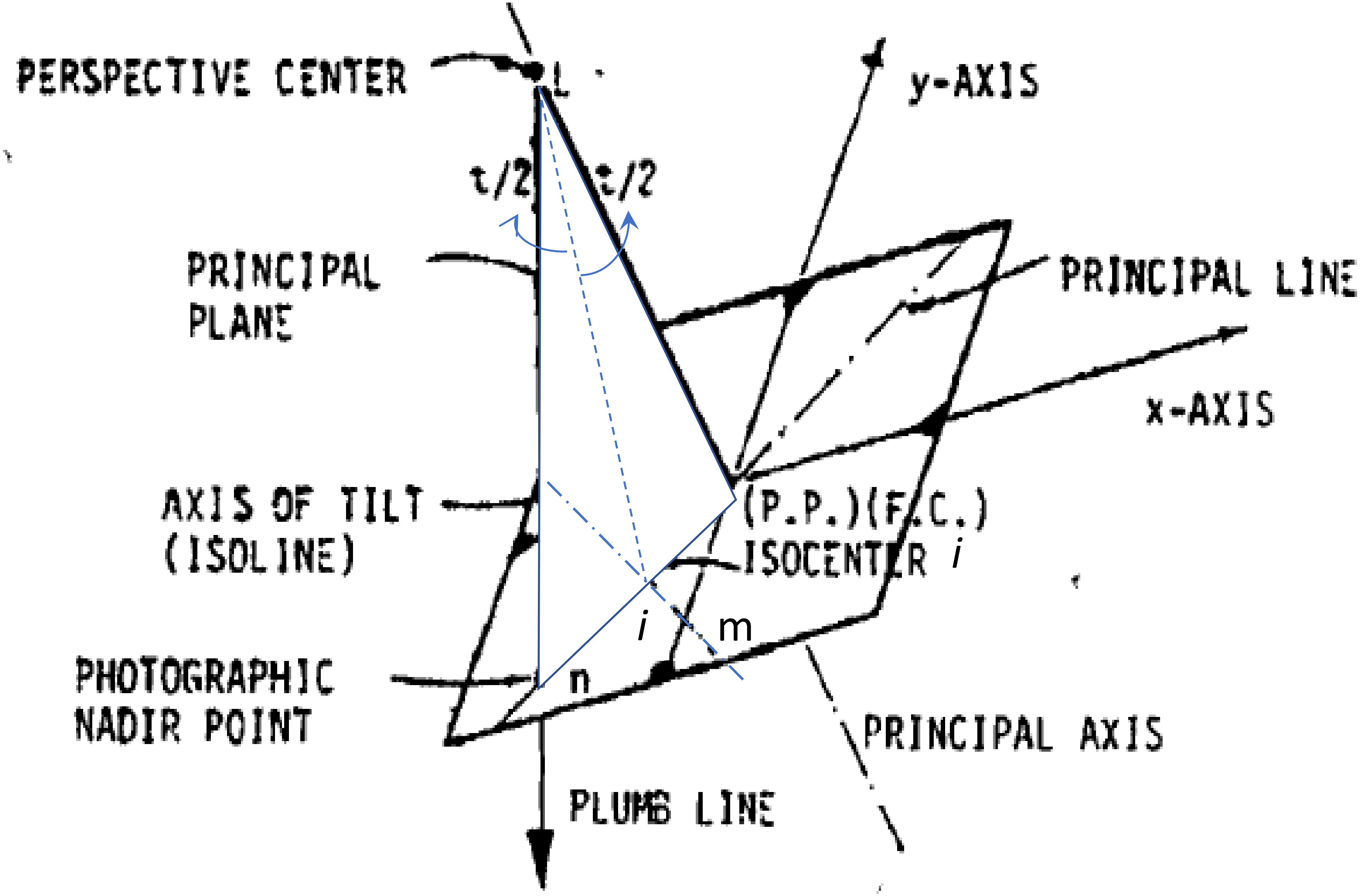
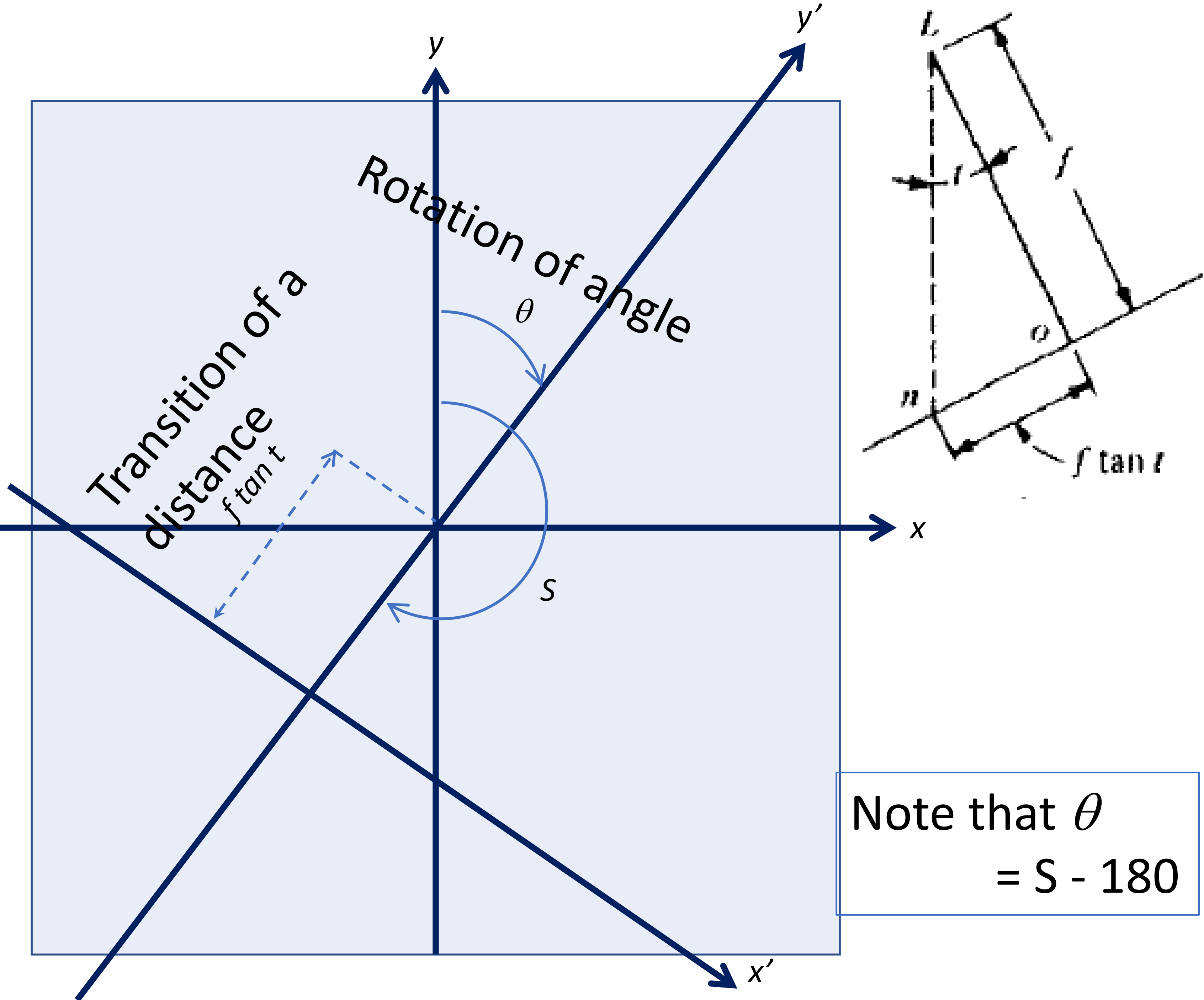


Figure (3-6) Basic elements of tilted photograph

What and why an auxiliary coordinate system?

- A step to relate photo coordinates to ground, because the photograph is tilted.
- Thus, photo and ground coordinates are not parallel any more.
- You need a system in between as a step to transfer photo coordinates to ground, specially that tilt is variable.

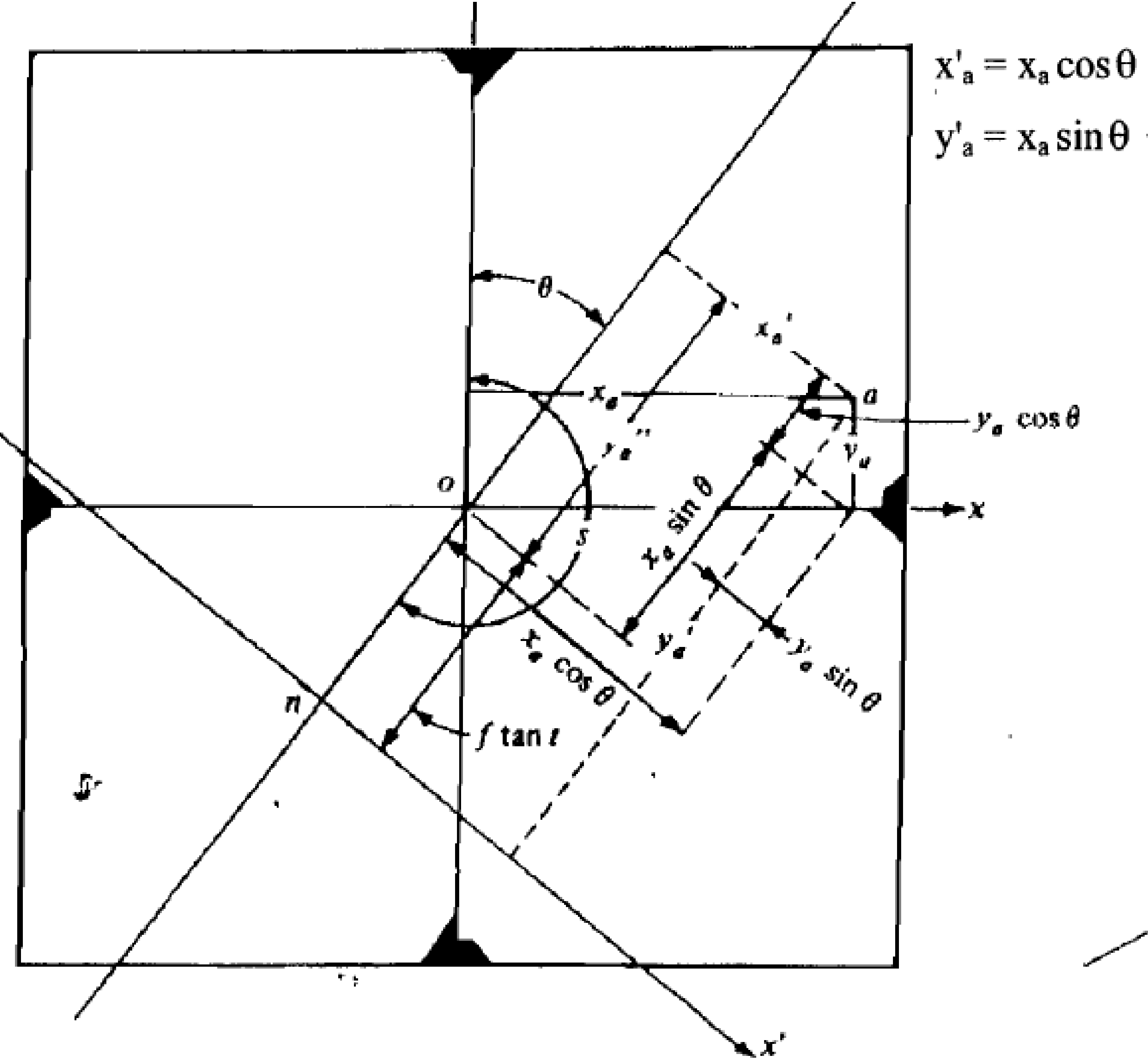
Transformation from photo coordinates (*x*, *y*) to an auxiliary coordinate system (x’, y’)



Note that θ = S - 180

$$x'_a = x_a \cos\theta - y_a \sin\theta$$

$$y'_a = x_a \sin\theta + y_a \cos\theta + f \tan t$$



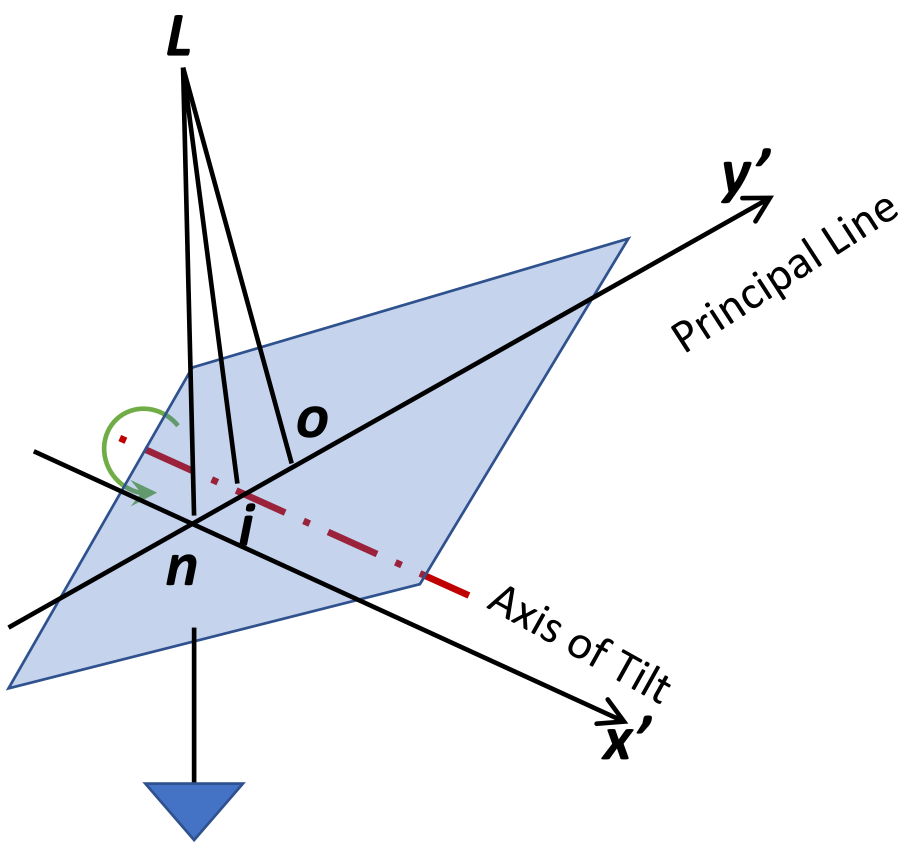
Relationship between Photo and Auxiliary coordinate system

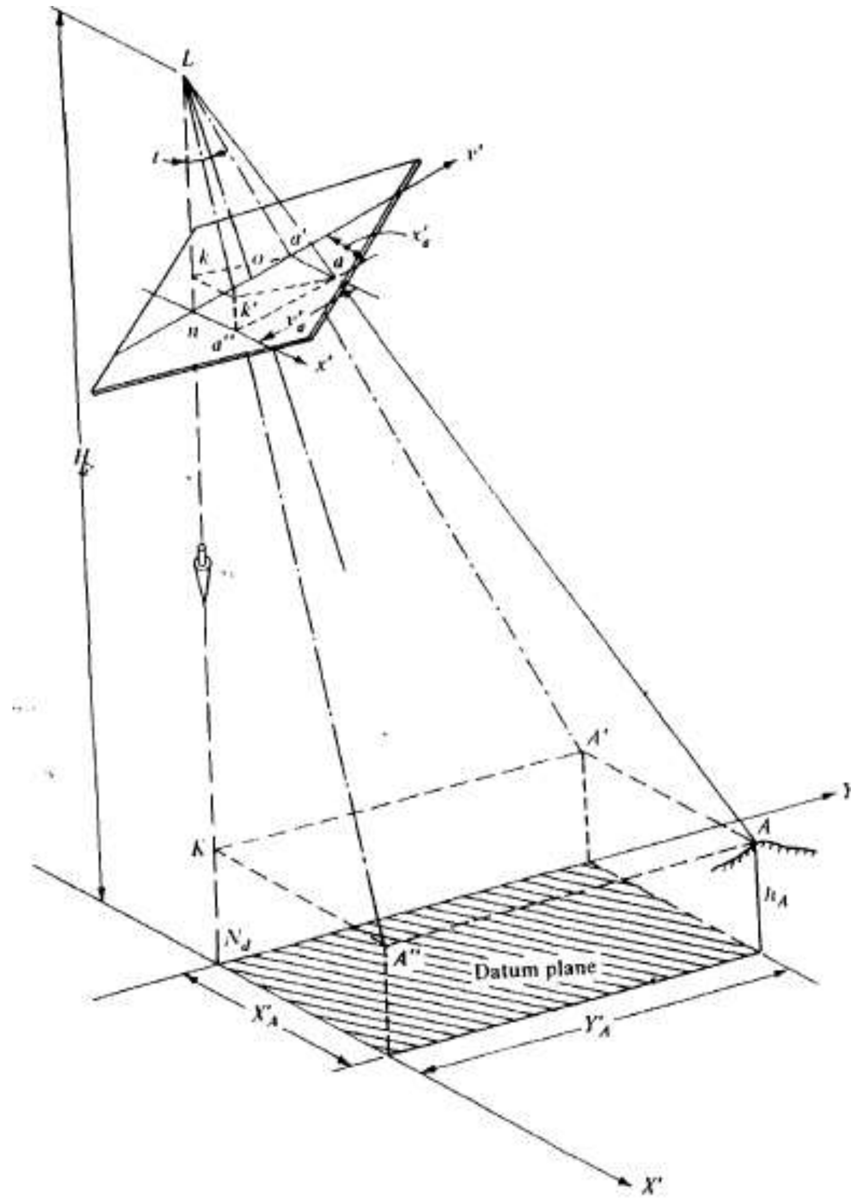
$$x'_a = x_a \cos\theta - y_a \sin\theta$$

$$y'_a = x_a \sin\theta + y_a \cos\theta + f \tan t$$

Scale of a tilted Photograph

The tilt of a photograph occurs around the axis of tilt in the direction of the principal line.

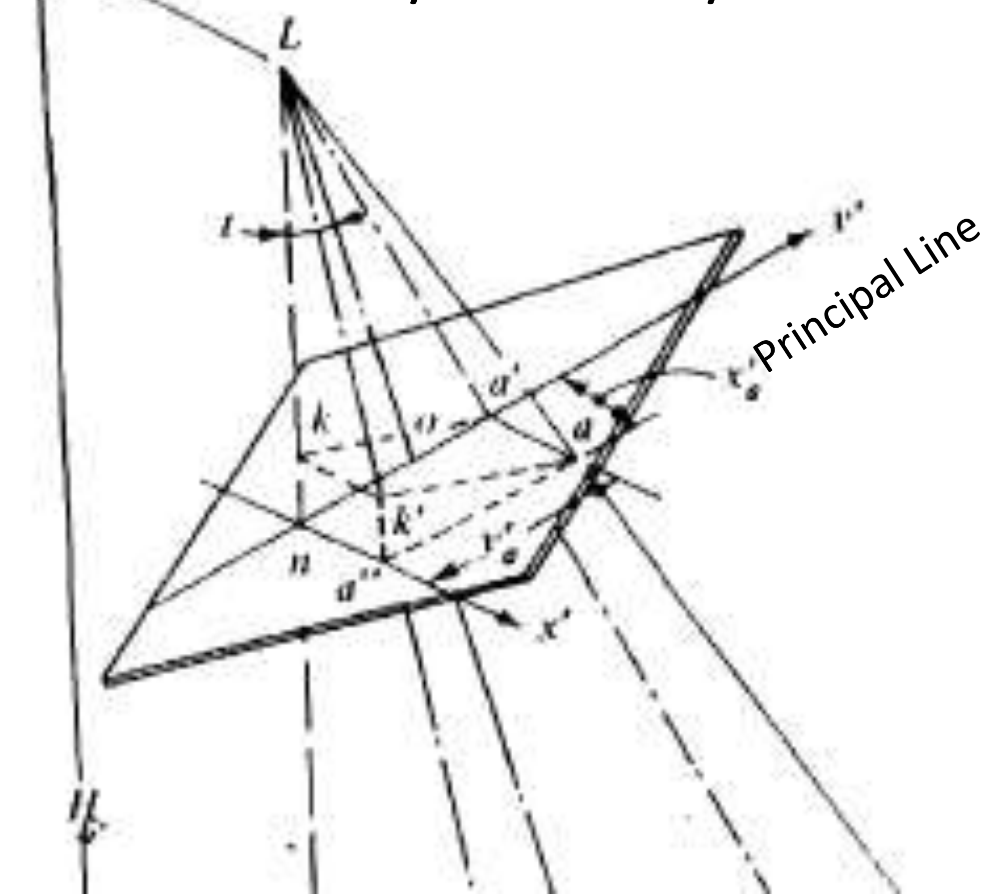


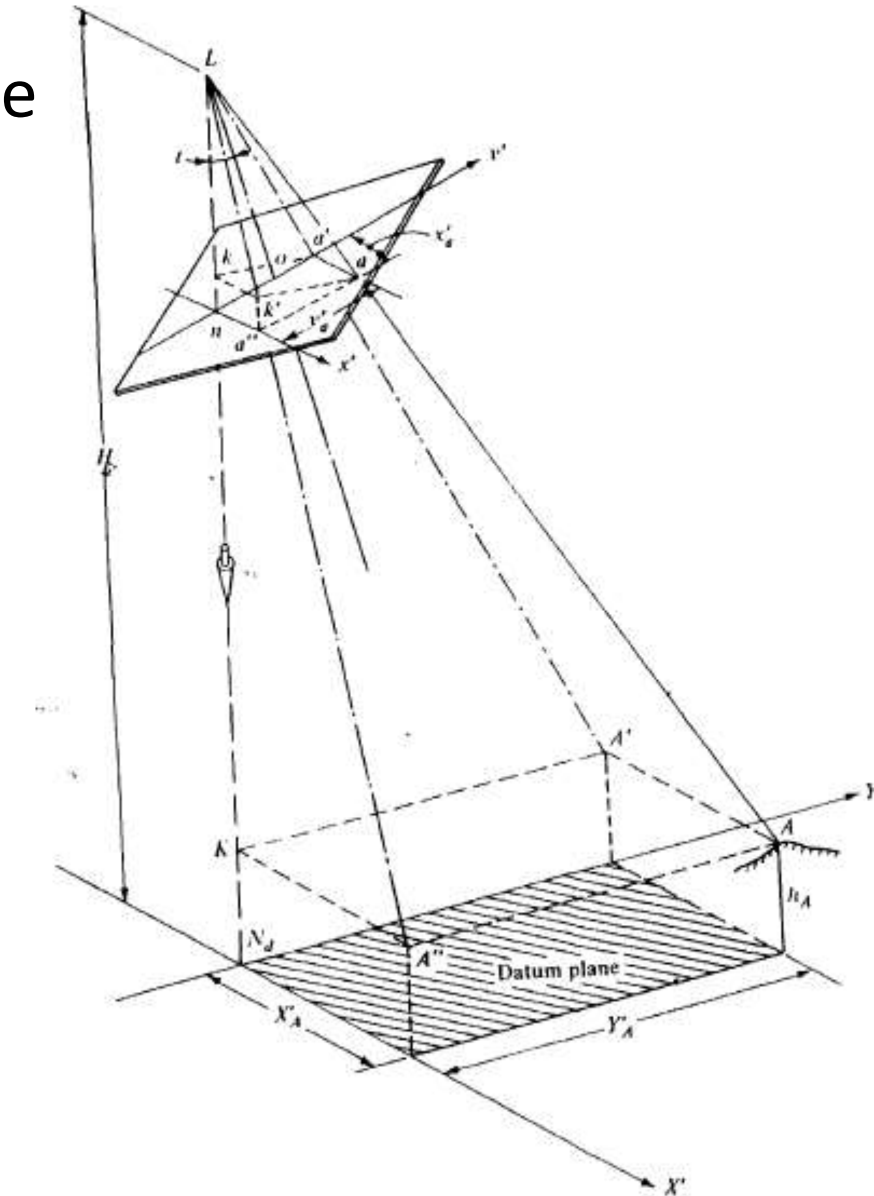


Scale of a tilted Photograph

- Scale = horizontal distance on the photo / horizontal distance on the ground =

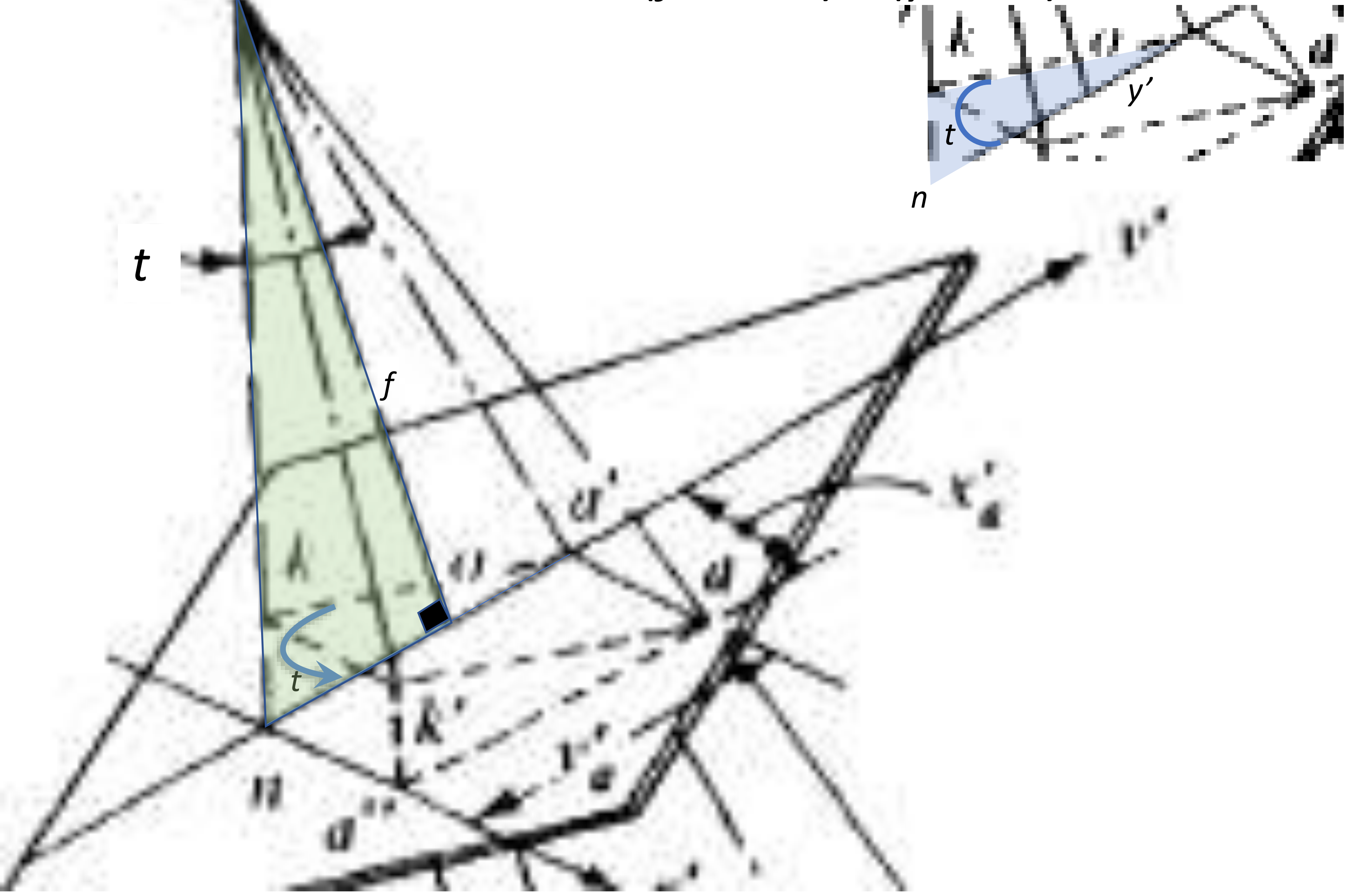
  $ka' / KA' = Lk / LK$





$S = Lk / LK$, but:

$Lk = Ln - nK = (f / \text{cost } t) - (y' \sin t)$, and

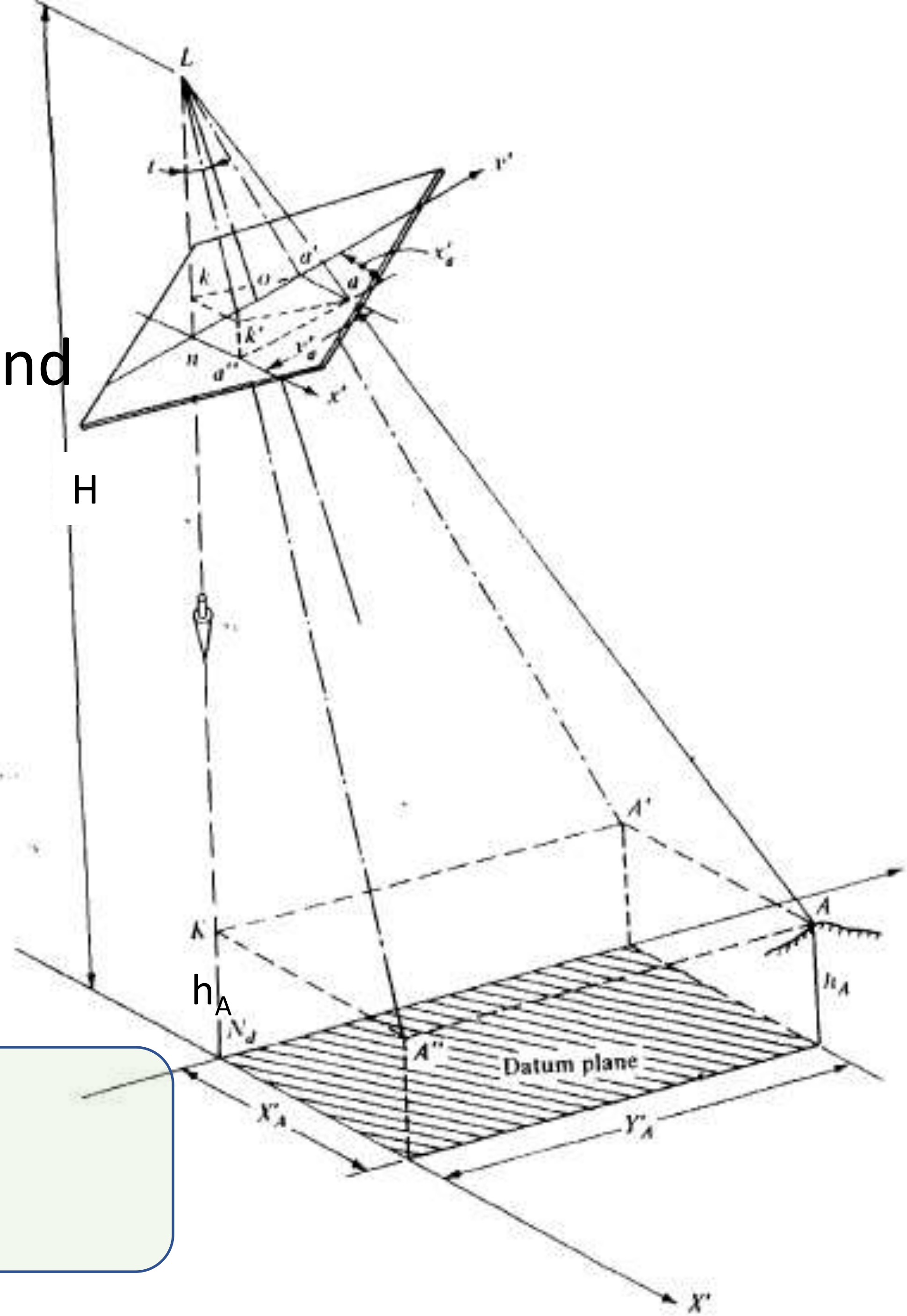


Also,

$LK = H - h_A$

$= $ flying height above ground

Then:



Scale of a tilted photograph

$S_A = f(\sec t) - y' \sin t \,/\, (H-h_A)$

Example

Example 3-1:

A tilted Photo is taken with a 6 inch focal length camera from a flying height of 8200 feet Tilt and swing angles are 3^{o} 30^{o} and 218^{o} respectively. Point (A) has an elevation of 1435 feet and its image coordinates are xa = -2 85 inch. ya 3.43 inch . What is the scale at point (a) ?

Solution

$\theta =$

Ground Coordinates from a tilted photograph

- Coordinates of point A in a ground coordinate system X', Y' where:
- X', Y' are parallel to *x'* and *y'* (auxiliary system)
- Ground Nadir N is the origin of the ground system
- Note that in the auxiliary coordinate system, lines parallel to *x'* are horizontal, thus *x'* on the photo is horizontal and directly related to ground X by the scale, or

$$X'_A = x' / S_A$$

- But in the auxiliary system, y' is in the direction of maximum tilt and not horizontal, the scale is ratio between horizontal projections.
- Ka: Horizontal projection of y' = $y' \cos t$
- Then,
- $Y' = y' \cos t / S$

Example

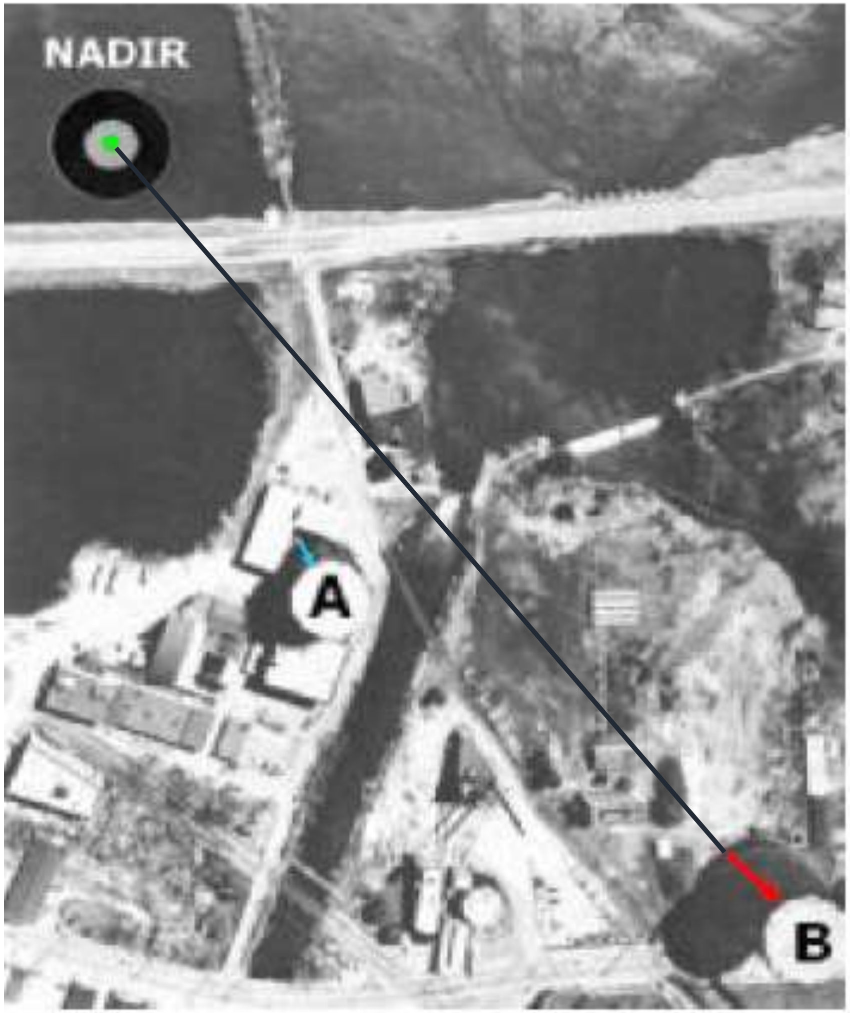
Example 3-1:

A tilted Photo is taken with a 6 inch focal length camera from a flying height of 8200 feet Tilt and swing angles are $3^0\ 30^0$ and 218^0 respectively. Point (A) has an elevation of 1435 feet and its image coordinates are xa = -2 85 inch. ya 3.43 inch . What is the scale at point (a) ?

If the image coordinates of another point (b) are xb = 3.09 inch, yb 1 .78 inch. and the elevation of (B) is 1587 feet .calculate ground coordinates of (A) and (B).

Relief Displacement of a Tilted Photograph

- Displacement of elevated points occurs from the nadir point *n* “intersection of vertical with the photo”.
- Since the tilt is small, the nadir *n* is close to the P. P. or *o*
- *The error can be ignored:*
- *Displacement is measured from o and the same equation applies.*
- *When will you NOT ignore that error????*



Tilt Displacement

- Important to learn since it provides basic knowledge needed for rectification.
- Rectification is the process of making equivalent vertical photographs from tilted photo
- An equivalent vertical photo is a photograph taken from the same exposure station L while the optical axis is vertical, with the same camera of focal length f
- For the geometry to be correct, the photograph should be tilted around the isoline, or the axis of tilt through *i, why??*

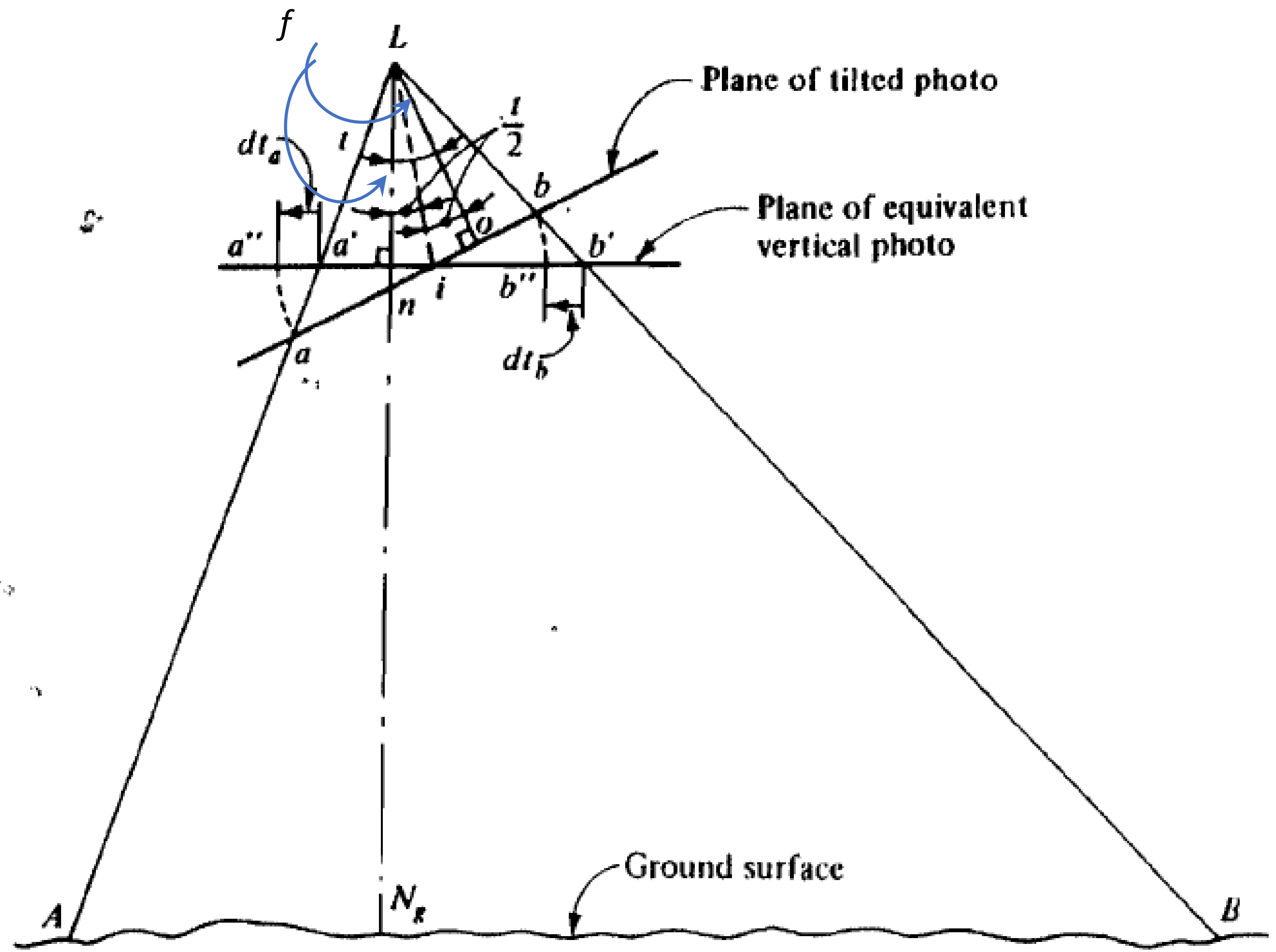


Figure (3-9) Tilt displacement in the principal plane of a tilted photograph

- Tilt displacement (d_t) is the distance by which the image of a point on a **vertical** photograph is shifted as the image is tilted.
- Assume that point B on the ground appears on the vertical photograph at point a' at a radial distance $r_{ia'}$ from the isoceneter. as the photograph is tilted, point A now appears at a on the tilted photograph at a distance r_{ia} from the isocenter i.
- The tilt displacement of a = r_{ia} - $r_{ia'}$
- Tilt displacement (from vertical to tilted photo) is inward (-ve) if the point is below the isoline such as a, and is outward (+ve) if the point is above the isoline such as b.
- Digital images can easily be rectified by shifting each pixel by (d_t) at a radial distance from the isocenter.

- Tilt displacement is calculated by the following equation:

$$d_t = \frac{(r_i)^2 \sin t \cos \lambda}{f - (r_i) \sin t \cos \lambda}$$

where:

	dt	is the amount of tilt displacement.
	r_i	is the radial distance from the isocenter to the image point.
and	λ	is the angle in the plane of photograph between the principal line (no) and the radial line r_i