

Geometry of Aerial Photographs

Definitions

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Photo Coordinate System

Defined by the fiducials in a film camera.
(x) axis is in direction of flight

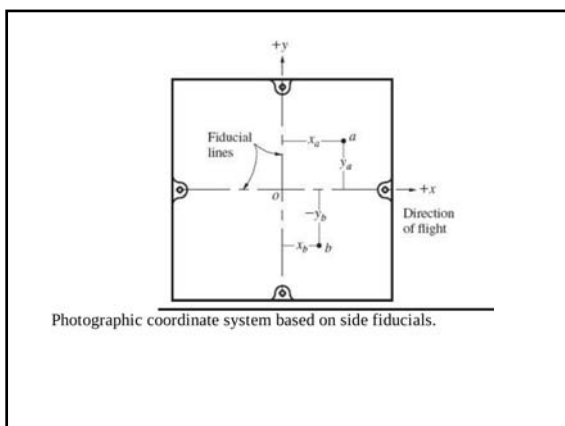
Focal length (F)

Principal point (a virtual cross-point of the dashed lines)
Focal length (principal distance)
Fiducial marks (printed in the image)
Photograph

Fiducial marks (printed in the image)
Principal point (a virtual cross-point of the dashed lines)

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Photo Coordinates (film)

Repeated slide

- We use positives for ease of geometry and familiarity of feature shapes, negatives may be used in certain applications
- Lines connecting middle **fiducials ON THE POSITIVE** define a photo coordinate system, in which x is in the direction of flight, A RIGHT-HAND coordinate system
- Measurements can be as accurate as 1 micron = 1/1000 mm

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Photo Coordinates (Digital)

(x, y) and (r, c)

Repeated slide

- In a digital image, we measure rows and column locations (r, c).
- In a digital camera, the relationship between pixel locations in (r, c) and photo (x, y) at center is defined within the camera, no need for additional transformation
- In a film camera, the relationship is between photo fiducials (x, y) and ground (X, Y).
- When we scan a photo taken by a film camera, it becomes in a digital format, we measure (r, c). In this case, we need to transform (r, c) that we measure to (x, y) photo coordinates to apply the equations.
- A two-dimensional coordinate transformation from (r, c) to (x, y) is needed if the camera is not digital.

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Geometry of aerial cameras

- Identify the following:
 - Photo Coordinate System (right-handed)
 - L: perspective center.
 - Fiducial Center F.C.: intersection of (x) and (y) axis as defined by the fiducials (in a film camera).
 - Principal Point (P.P) or O: the point where the perpendicular from the perspective center intersects the photograph. Usually deviates from the F.C by a very small distance.
 - Principal axis: the line perpendicular from the principal center on the plane of the photograph (negative).
 - f: the focal length, equals the Principal Distance.

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Geometry of Vertical Photographs

Repeated slide

- Define: image coordinate system (right handed), principal point, exposure station.
- If a film is used, measurements could be done using negatives or diapositives, same geometry, referred to the fiducials
- In digital images, row and columns of pixels define a coordinate system

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Geometry of aerial cameras

Repeated slide

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Figure 3.8 Geometry of vertical photographs.

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Geometry of a digital frame camera

- Similar geometry is assumed in case of a digital camera
- Uses a two-dimensional array of CCD elements mounted at the focal plane of the camera.
- The image is a grid of picture elements (pixels)

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Vertical Photographs

Single Photo Applications Scale

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Scale of a Vertical Photograph

- Scale of a photograph is the ratio of a distance on a photo to the same distance on the ground.
- Differences between photographs and maps:
 - Projection: orthogonal or perspective
 - Scale constant on maps, but varies on photos. The closer the object to the camera (higher), the bigger it looks.

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Orthophoto maps

- geometrically corrected images, can be used as maps.
- The shift in image is proportional to the relief "height" of location in a photo.
- A DTM is needed to correct "rectify" the image so that images are displaced to their corresponding location on maps.

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Orthophotos and True Orthophotos

- Orthophotos are corrected for relief of topography, top of tall structures are still shifted.
- True orthophotos are produced from multiple images to eliminate the displacement of the top of tall objects.

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- Scale of a photograph is the ratio of a distance on a photo to the same distance on the ground.

The scale of a vertical photograph approximately equals to the ratio of the flying height above the ground and the focal length of the camera lens (assuming horizontal flat terrain)

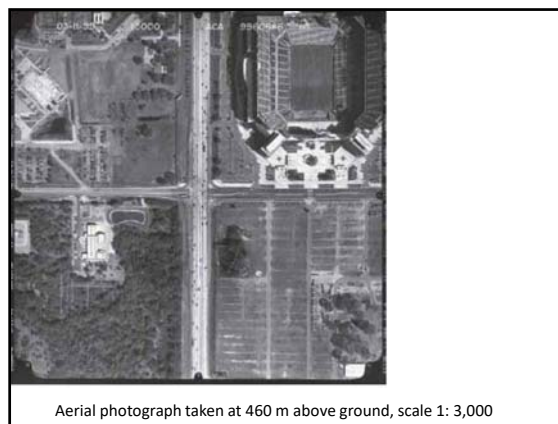
$$Scale = \frac{imageDist}{surfaceDist} = \frac{f}{H-h}$$

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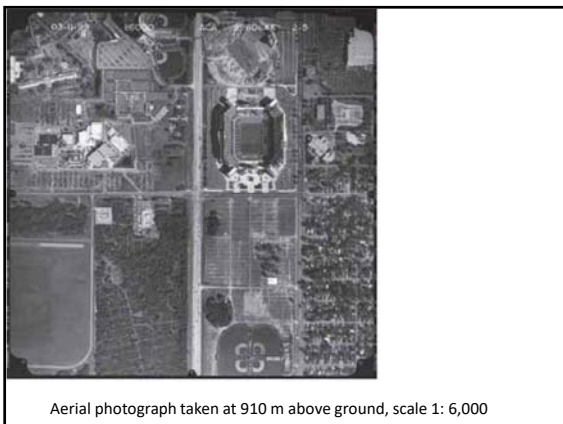
- Scale of a vertical photo of flat horizontal ground is:

$$S = f / (H-h)$$

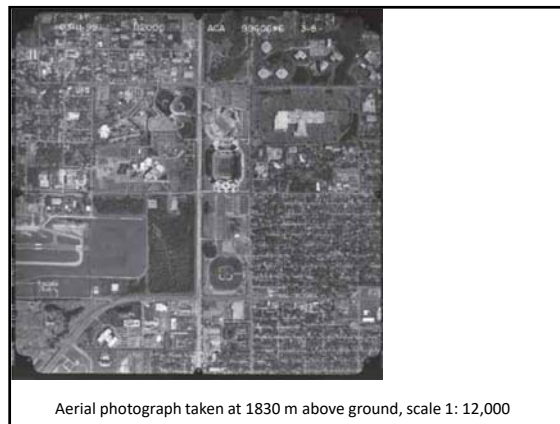
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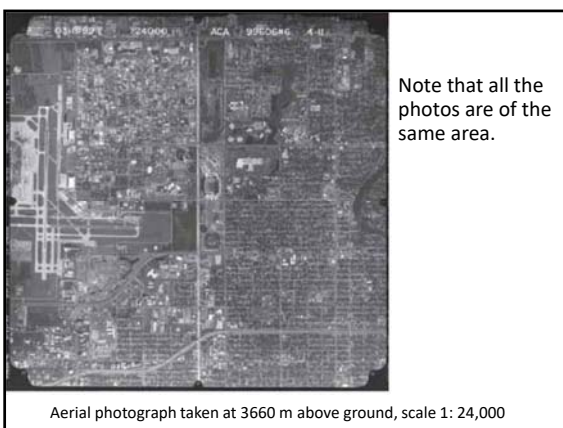
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Example

A vertical aerial photograph is taken over flat terrain with a 152.4 mm-focal-length camera from an altitude of 1830 m above ground. What is the photo scale?

- Answer:

$$S = 152.4 \text{ mm} / 1830 \text{ m}$$

$$= 0.1524 \text{ m} / 1830 \text{ m} =$$

$$1: 12,000$$

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Scale of a vertical photograph over variable terrain

But, ground is not always flat and horizontal. H in that case, elevation of ground, is variable, how do we define a scale??

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Scale of a Vertical Photograph

- Definitions:
 - f: focal length of the camera
 - H: flying height above datum (MSL?)
 - h: flying height above ground
- Scale (s) at any point:

$$S = \frac{f}{H - h}$$
- Average scale of a photograph:

$$S_{avg} = \frac{f}{H - h_{avg}}$$
- If f, H, and h are not available, but a map is available then:

$$\text{Photo Scale} = \frac{\text{photo distance}}{\text{map distance}} \times \text{map scale}$$

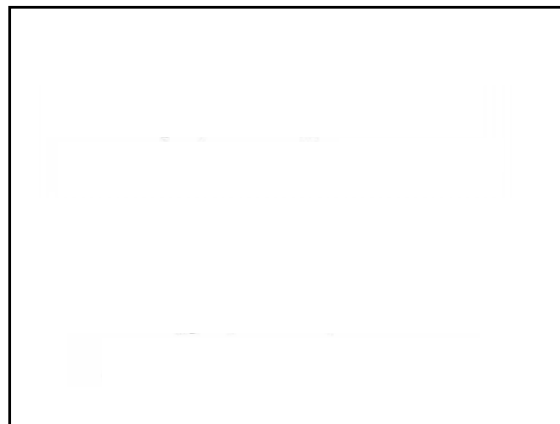
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Example

Suppose that highest terrain h_1 , average terrain h_{avg} , and lowest terrain h_2 of Fig. 6-3 are 610, 460, and 310 m above mean sea level, respectively. Calculate the maximum scale, minimum scale, and average scale if the flying height above mean sea level is 3000 m and the camera focal length is 152.4 mm.

Answer:

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Example

On a vertical photograph the length of an airport runway measures 160 mm. On a map that is plotted at a scale of 1:24,000, the runway is measured as 103 mm. What is the scale of the photograph at runway elevation?

Answer

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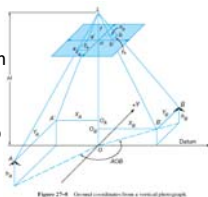
Vertical Photographs

Single Photo Applications
Local Ground Coordinates

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Ground Coordinates from a Single Vertical Photograph

- With a local image coordinate system defined, we define an arbitrary ground coordinate system.
- That ground system could be used to compute distances and azimuths. Coordinates can also be transformed to any system
- In that ground system:
 - $X_a = x_a \cdot a$ (photograph scale at a)
 - $Y_a = y_a \cdot a$ (photograph scale at a)



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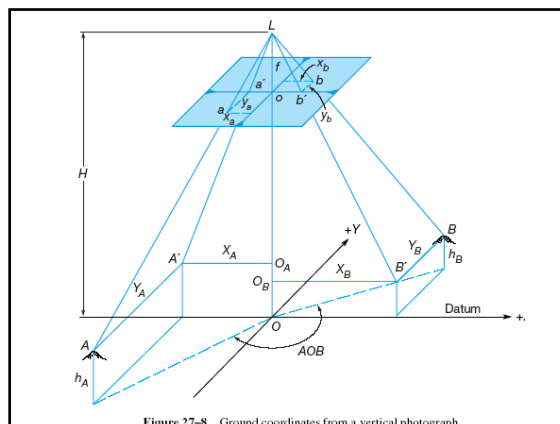


Figure 27-8 Ground coordinates from a vertical photograph.

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Example (consider answering on your own)

A vertical photograph was taken with a camera having a focal length of 152.3 mm. Ground points A and B have elevations 437.4 m and 445.3 m above sea level, respectively, and the horizontal length of line AB is 584.9 m. The images of A and B appear at a and b, and their measured photo coordinates are $x_a = 18.21$ mm, $y_a = -61.32$ mm, $x_b = 109.65$ mm, and $y_b = -21.21$ mm. Calculate the flying height of the photograph above sea level.

Answer:

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Vertical Photographs

Single Photo Applications
Relief displacement

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Relief displacement

Towers A and B are equally high, but placed at different distances from the nadir point, thus have different relief displacements. A tower, depicted beneath nadir point has no relief displacement

Aerial photo

Earth surface

Nadir point A B

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Relief displacement from Nadir (enlarged)

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Relief displacement from Nadir (Center)

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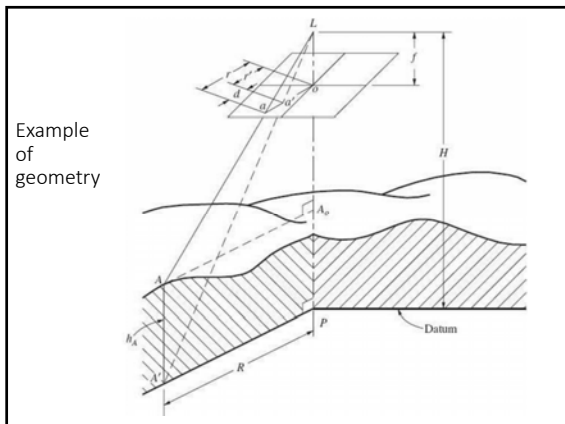
Relief Displacement on a Vertical Photograph

الازاحه بسبب الارتفاع

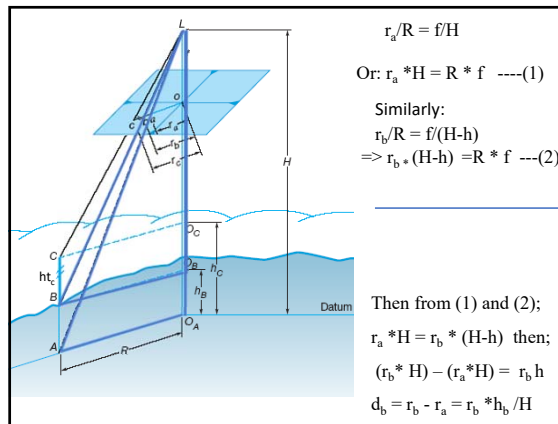
The shift of an image from its location as caused by the object's relief. Two points on a vertical line will appear as one line on a map, but two points, usually, on a photograph. الازاحه لنقطه على الصوره من مكانها بسبب الارتفاع

- In a vertical photo, the displacement is from the principal point.

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• Relief displacement (d) of a point with respect to a point on the datum :

$$d = \frac{r h}{H}$$

where:

r : is the radial distance on the photo to the high point
 h : elevation of the high point, and H is flying height above datum

• Assuming that the datum is at the bottom of vertical object, H is the flying height above ground, the value h will compute the object height.

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Also:

$$r_c/R = f / \{(H-h) - ht_c\}$$

$$\Rightarrow r_c * \{(H-h) - ht_c\} = R * f$$
 ---(3)

From (2) and (3):

$$r_c * (H-h) - (r_c * ht_c) = r_b * (H-h)$$

$$(r_c - r_b) * (H-h) = r_c * ht_c$$

$$d = r_c - r_b = r_c * ht_c / (H-h)$$

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In general:

Assume that point C is vertically above B, they are shown on the photograph as (c) and (b). Measured radial distances from the center to points c and b (r_c and r_b), then

$$d_c = r_c - r_b \quad \text{and};$$

$$d_c = (r_c * ht_c) / (\text{flying height above ground} = H - h_b)$$

Note that relief displacement is eliminated in true ortho photos

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Example:

A vertical photograph taken from an elevation of 535 m above mean sea level (MSL) contains an image of a tall vertical radio tower. The elevation at the base of the tower is 259 m above MSL. The relief displacement d of the tower was measured as 54.1 mm. What is the height of the tower?

Answer:

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Vertical Photographs

*Single Photo Applications
Flying Height*

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Flying Height of a Vertical Photograph

- Flying height can be determined by:
 - Readings on the photos
 - Applying scale equation, if scale can be computed
 - Example: what is the flying height above datum if $f=6"$, average elevation of ground is 900ft, scale is $1":100ft$? Is it 1500'?
 - Or, if two control points appear in the photograph, solve the equation:

$$L^2 = (X_B - X_A)^2 + (Y_B - Y_A)^2$$
 then solve the same equation again replacing the ground coordinates with the photo coordinates. Get the scale.

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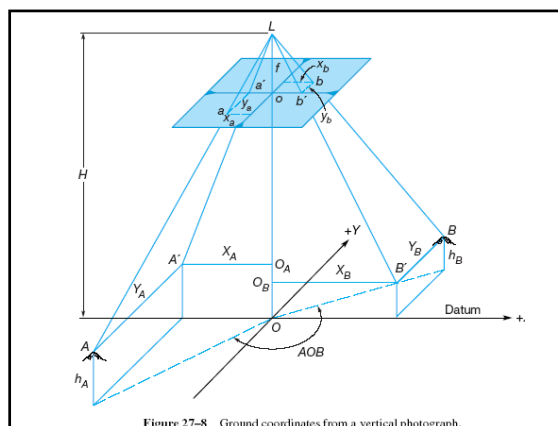
Ground Coordinates from a Single Vertical Photograph

- With image coordinate system defined, we may define an arbitrary ground coordinate system parallel to (x,y) origin at nadir.
- That ground system could be used to compute distances and azimuths. Coordinates can also be transformed to any system
- In that ground system:

$$X_a = x_a * (\text{photograph scale at } a)$$

$$Y_a = y_a * (\text{photograph scale at } a)$$

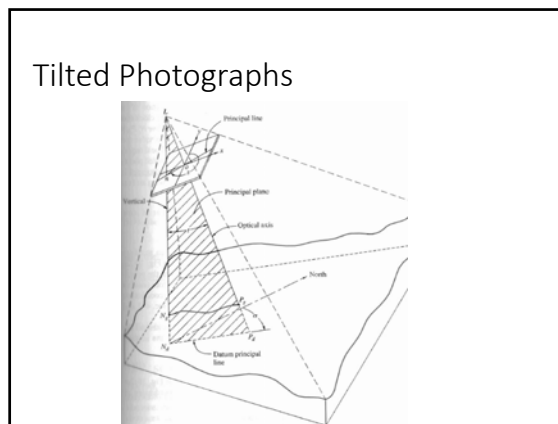
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Tilted Photographs

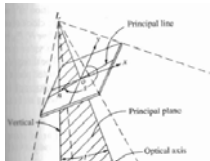
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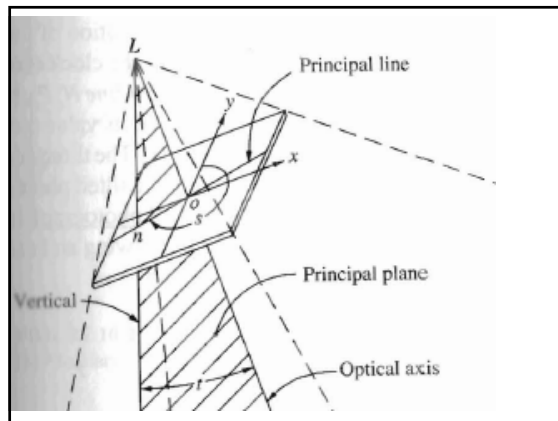
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Basic elements of a tilted photographs

- The optical axis is tilted from the vertical
- Identify the following:
 - t = angle of tilt between the plumb line and the optical axis LO
 - i = the isocenter: the line bisecting the tilt angle intersects the principal line in the isocenter.
 - n_o = the principal line joining the nadir point (n) and the principal point (o).



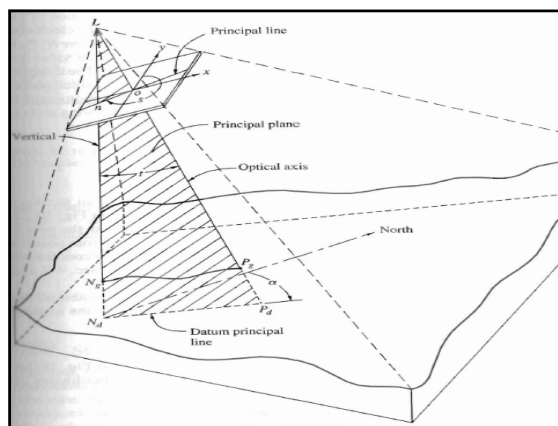
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- Lno = the principal plane: it is the vertical plane containing o , L and n (shaped plane).
- im = axis of tilt: it is the line perpendicular to the principal line from the isocenter i in the plane of the photograph.
- S = the swing angle: it is the angle measured from the positive photographic y-axis clockwise to the principal line (on).
- $x'y'$ axes are the auxiliary coordinate system of the tilted photograph where:
 - y' is the principal line (n_o).
 - x' is the perpendicular to y' from point n .
 - θ = the rotation angle between y and y' axes in a counterclockwise direction.

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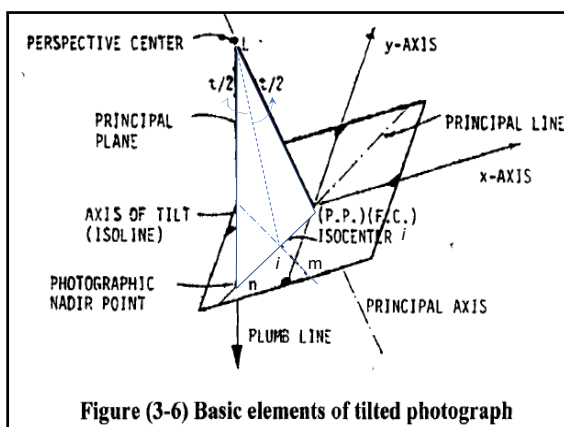


Figure (3-6) Basic elements of tilted photograph

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What and why an auxiliary coordinate system?

- A step to relate photo coordinates to ground, because the photograph is tilted.
- Thus, photo and ground coordinates are not parallel any more.
- You need a system in between as a step to transfer photo coordinates to ground, specially that tilt is variable.

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